Quantum attack against some candidate obfuscators based on GGH13

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What is this talk about

Quantum attack against some candidate obfuscators built upon the GGH13 multilinear map [GGH13a]

[GGH13a] S. Garg, C. Gentry and S. Halevi. Candidate multilinear maps from ideal lattices, Eurocrypt.

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Quantum attack against some candidate obfuscators built upon the GGH13 multilinear map [GGH13a]

- GGH13 is known to be weak in quantum world
- Transform this weakness into concrete attack on obfuscators
- Nothing quantum in this talk

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An obfuscator O for a class of circuits $\mathcal{C}$ is an efficiently computable function over $\mathcal{C}$ such that

$$\forall C \in \mathcal{C}, \forall x, C(x) = O(C)(x)$$

In this talk, $\mathcal{C}$ = polynomial size circuits

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- iO: $\forall C_1 \equiv C_2$, i.e. $C_1(x) = C_2(x)\ \forall x$,

$$O(C_1) \simeq_c O(C_2)$$

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2 Many cryptographic constructions from iO: functional encryption, deniable encryption, NIKZs, oblivious transfer, ...

Multilinear maps (mmaps) and iO

Observation

Almost all iO constructions for all circuits rely on multilinear maps (mmaps)

Three main candidate multilinear maps: GGH13, CLT13, GGH15

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In this talk: we exploit known weakness of GGH13 to mount concrete attacks against some iO using it.

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2018: [CHKL18], attack against all obfuscators, for specific choices of parameters

State of the art and contribution

iO (using GGH13) / Attacks	Branching program obfuscators				Circuit obfuscators
	[GGH+13b]	[BR14]	[AGIS14, MSW14] [PST14, BGK+14] [BMSZ16]	[GMM+16]	[Zim15, AB15] [DGG+16]
[MSZ16]		✓	✓		
[CGH17]*	✓				
[CHKL18]†	✓	✓	✓	✓	
This talk‡			✓	✓	✓

* for input-partitionable branching programs
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Attacks \ iO (using GGH13)	Branching program obfuscators				Circuit obfuscators
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[MSZ16]		✓	✓		
[CGH17]⋆	✓				
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Outline of the talk

1. Simple obfuscator
2. The attack

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- 2ℓ matrices $A_{i,b}$ (for $i \in \{1, \ldots, \ell\}$ and $b \in \{0, 1\}$),
- two vectors A_0 and $A_{\ell+1}$,
- a function inp : $\{1, \ldots, \ell\} \to \{1, \ldots, r\}$ (where r is the size of the input).

i	1	2	3	4	5	6
inp(i)	1	1	2	1	3	2

$x = 0 \quad 1 \quad 1$

A_0 $\quad$ $A_{1,1}$, $A_{1,0}$ $\quad$ $A_{2,1}$, $A_{2,0}$ $\quad$ $A_{3,1}$, $A_{3,0}$ $\quad$ $A_{4,1}$, $A_{4,0}$ $\quad$ $A_{5,1}$, $A_{5,0}$ $\quad$ $A_{6,1}$, $A_{6,0}$ $\quad$ A_7

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Cryptographic multilinear maps

Definition: κ-multilinear map

Different levels of encodings, from 1 to κ.
Denote by $\mathrm{Enc}(a, i)$ a level-i encoding of the message a.

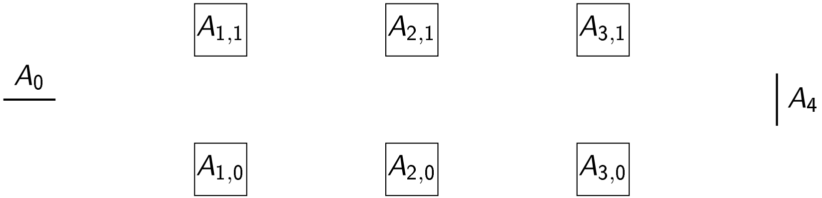
Addition: $\mathrm{Add}(\mathrm{Enc}(a_1, i), \mathrm{Enc}(a_2, i)) = \mathrm{Enc}(a_1 + a_2, i)$.

Multiplication: $\mathrm{Mult}(\mathrm{Enc}(a_1, i), \mathrm{Enc}(a_2, j)) = \mathrm{Enc}(a_1 \cdot a_2, i + j)$.

Zero-test: $\text{Zero-test}(\mathrm{Enc}(a, \kappa)) = \text{True iff } a = 0$.

Simple obfuscator

- **Input:** A branching program
- Randomize the branching program
  - Add random diagonal blocks
  - Killian's randomization
  - Multiply by random (non zero) bundling scalars
- Encode the matrices using GGH13
- **Output:** The encoded matrices and vectors



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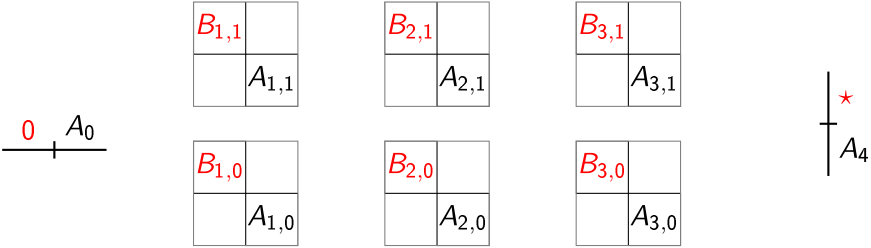

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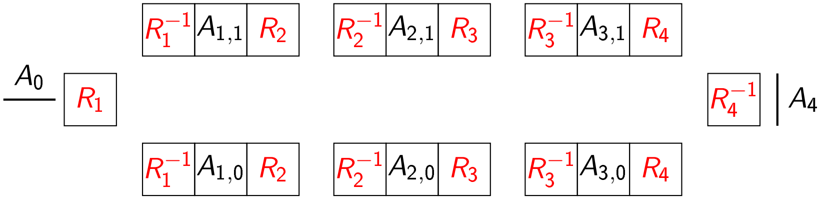

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$$\underline{A_0} \qquad\qquad\qquad\qquad\qquad\qquad \Big| A_4$$

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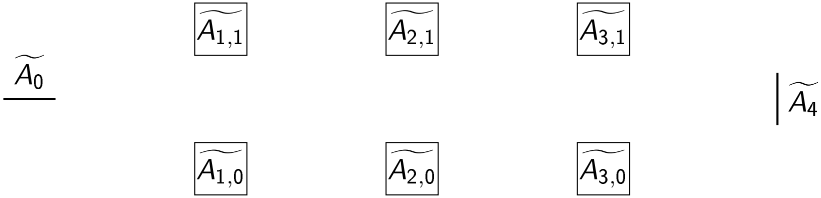

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$\mathrm{Enc}(\widetilde{A_{1,1}})$ $\mathrm{Enc}(\widetilde{A_{2,1}})$ $\mathrm{Enc}(\widetilde{A_{3,1}})$

$\mathrm{Enc}(\widetilde{A_0})$ $\mathrm{Enc}(\widetilde{A_4})$

$\mathrm{Enc}(\widetilde{A_{1,0}})$ $\mathrm{Enc}(\widetilde{A_{2,0}})$ $\mathrm{Enc}(\widetilde{A_{3,0}})$

Outline of the talk

GGH13 in a quantum world

Reminder: κ-multilinear map

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Zero-test: $\text{Zero-test}(\mathrm{Enc}(a, \kappa)) = \text{True iff } a = 0 \mod p$.

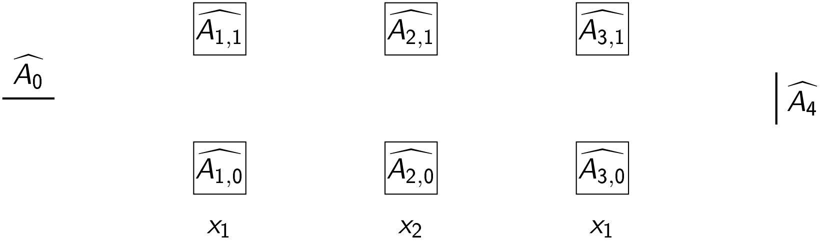
With a quantum computer

$$\text{Double-zero-test}(\mathrm{Enc}(a, 2\kappa)) = \text{True iff } a = 0 \mod p^2$$

Mixed-input attack

Notations

- $A_{i,b}$ input branching program
- $\widetilde{A_{i,b}}$ after randomisation
- $\widehat{A_{i,b}}$ after encoding with GGH13 map (output of the iO)



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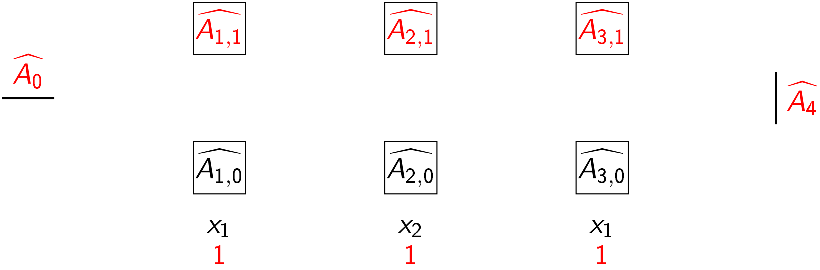

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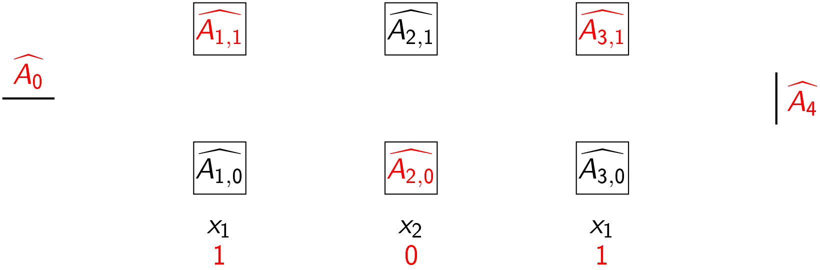

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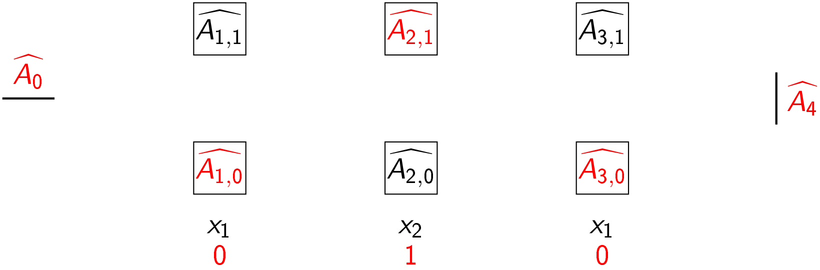

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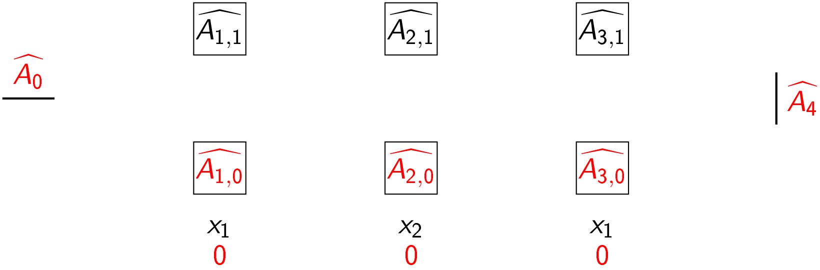

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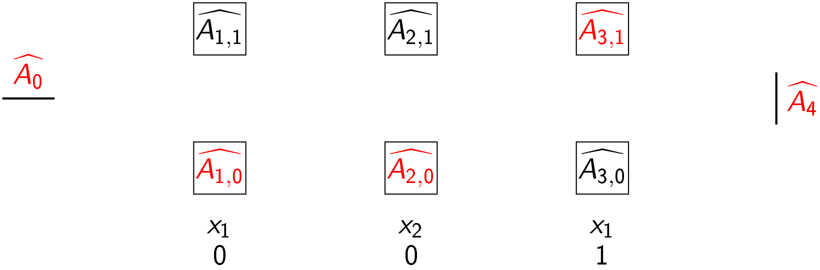

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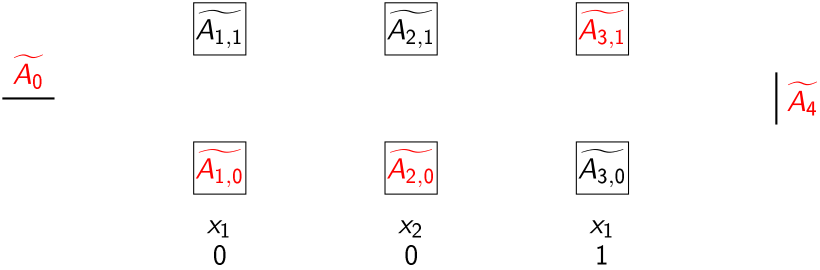

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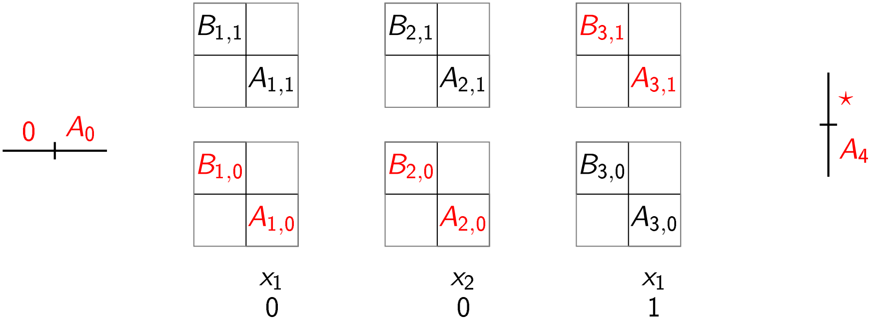

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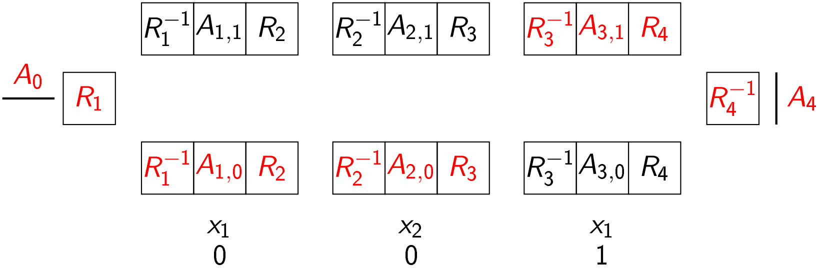

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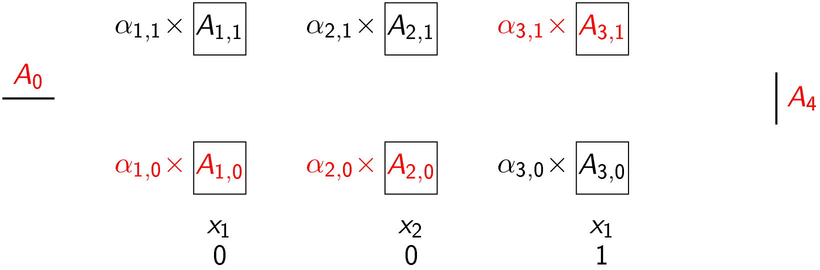

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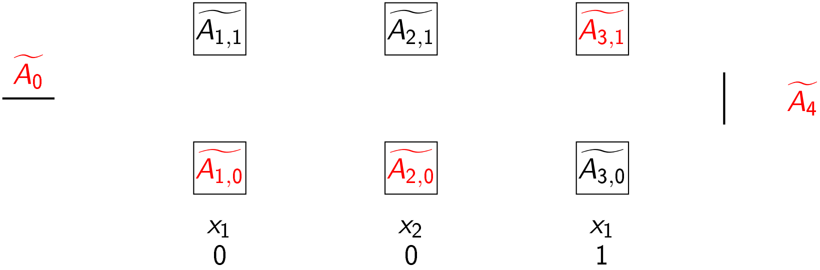

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$$
\underline{\mathrm{Enc}(\widetilde{A_0},1)} \quad
\begin{matrix}
\mathrm{Enc}(\boxed{\widetilde{A_{1,1}}},1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,1}}},1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,1}}},1) \\
\mathrm{Enc}(\boxed{\widetilde{A_{1,0}}},1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,0}}},1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,0}}},1) \\
x_1 & x_2 & x_1 \\
0 & 0 & 1
\end{matrix}
\quad \Big| \, \mathrm{Enc}(\widetilde{A_4},1)
$$

Preventing mixed-input attacks

- In the randomization phase $\Rightarrow$ not in this talk

Preventing mixed-input attacks

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Preventing mixed-input attacks

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Mmap degree: $\kappa = 5$

$$\mathrm{Enc}(\widetilde{A_0},1) \quad \begin{matrix} \mathrm{Enc}(\boxed{\widetilde{A_{1,1}}},1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,1}}},1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,1}}},1) \\ \mathrm{Enc}(\boxed{\widetilde{A_{1,0}}},1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,0}}},1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,0}}},1) \\ x_1 & x_2 & x_1 \end{matrix} \quad \Big| \mathrm{Enc}(\widetilde{A_4},1)$$

Preventing mixed-input attacks

- In the randomization phase ⇒ not in this talk
- Using the mmap ⇒ straddling set system

Mmap degree: $\kappa = 6$

$$\underline{\mathrm{Enc}(\widetilde{A_0}, 1)} \quad \begin{matrix} \mathrm{Enc}(\boxed{\widetilde{A_{1,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,1}}}, 2) \\ \mathrm{Enc}(\boxed{\widetilde{A_{1,0}}}, 2) & \mathrm{Enc}(\boxed{\widetilde{A_{2,0}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,0}}}, 1) \\ x_1 & x_2 & x_1 \end{matrix} \quad \Big| \mathrm{Enc}(\widetilde{A_4}, 1)$$

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$\mathrm{Enc}(\widetilde{A_{1,1}}, 1)$ $\mathrm{Enc}(\widetilde{A_{2,1}}, 1)$ $\mathrm{Enc}(\widetilde{A_{3,1}}, 2)$

$\mathrm{Enc}(\widetilde{A_0}, 1)$ $\mathrm{Enc}(\widetilde{A_4}, 1)$

$\mathrm{Enc}(\widetilde{A_{1,0}}, 2)$ $\mathrm{Enc}(\widetilde{A_{2,0}}, 1)$ $\mathrm{Enc}(\widetilde{A_{3,0}}, 1)$

x_1 x_2 x_1

0 0 1

Preventing mixed-input attacks

- In the randomization phase $\Rightarrow$ not in this talk
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Mmap degree: $\kappa = 6$

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Total level: 7 $\Rightarrow$ cannot zero-test

Attack idea: double mixed input

Reminder

In quantum world, we have

$$\text{Double-zero-test}(\text{Enc}(a, 2\kappa)) = \text{True iff } a = 0 \mod p^2$$

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	x_1	x_2	x_1
1	$\text{Enc}(\boxed{\widetilde{A_{1,1}}}, 1)$	$\text{Enc}(\boxed{\widetilde{A_{2,1}}}, 1)$	$\text{Enc}(\boxed{\widetilde{A_{3,1}}}, 2)$
0	$\text{Enc}(\boxed{\widetilde{A_{1,0}}}, 2)$	$\text{Enc}(\boxed{\widetilde{A_{2,0}}}, 1)$	$\text{Enc}(\boxed{\widetilde{A_{3,0}}}, 1)$

$\text{Enc}(\widetilde{A_4}, 1)$ $\Rightarrow$ Level 7

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$\text{Enc}(\widetilde{A_4}, 1) \Rightarrow$ Level 7

$\text{Enc}(\widetilde{A_0}, 1)$

	x_1	x_2	x_1
1	$\text{Enc}(\boxed{\widetilde{A_{1,1}}}, 1)$	$\text{Enc}(\boxed{\widetilde{A_{2,1}}}, 1)$	$\text{Enc}(\boxed{\widetilde{A_{3,1}}}, 2)$
0	$\text{Enc}(\boxed{\widetilde{A_{1,0}}}, 2)$	$\text{Enc}(\boxed{\widetilde{A_{2,0}}}, 1)$	$\text{Enc}(\boxed{\widetilde{A_{3,0}}}, 1)$

$\text{Enc}(\widetilde{A_4}, 1) \Rightarrow$ Level 5

Attack idea: double mixed input

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In quantum world, we have

$$\text{Double-zero-test}(\text{Enc}(a, 2\kappa)) = \text{True iff } a = 0 \mod p^2$$

$\underline{\text{Enc}(\widetilde{A_0}}, 1)$ $\quad$ $\text{Enc}(\boxed{\widetilde{A_{1,1}}}, 1)$ $\text{Enc}(\boxed{\widetilde{A_{2,1}}}, 1)$ $\text{Enc}(\boxed{\widetilde{A_{3,1}}}, 2)$ $\quad$ $|\ \text{Enc}(\widetilde{A_4}, 1)$ $\Rightarrow$ Level 7

$\text{Enc}(\boxed{\widetilde{A_{1,0}}}, 2)$ $\text{Enc}(\boxed{\widetilde{A_{2,0}}}, 1)$ $\text{Enc}(\boxed{\widetilde{A_{3,0}}}, 1)$

x_1 $\quad$ x_2 $\quad$ x_1

$\times$ $\quad$ $\underline{\text{Enc}(\widetilde{A_0}}, 1)$ $\quad$ $\text{Enc}(\boxed{\widetilde{A_{1,1}}}, 1)$ $\text{Enc}(\boxed{\widetilde{A_{2,1}}}, 1)$ $\text{Enc}(\boxed{\widetilde{A_{3,1}}}, 2)$ $\quad$ $|\ \text{Enc}(\widetilde{A_4}, 1)$ $\Rightarrow$ Level 5

$\text{Enc}(\boxed{\widetilde{A_{1,0}}}, 2)$ $\text{Enc}(\boxed{\widetilde{A_{2,0}}}, 1)$ $\text{Enc}(\boxed{\widetilde{A_{3,0}}}, 1)$

x_1 $\quad$ x_2 $\quad$ x_1

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x_1 x_2 x_1

$\text{Enc}(\widetilde{A_4}, 1)$ $\Rightarrow$ Level 7

× $\text{Enc}(\widetilde{A_0}, 1)$

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x_1 x_2 x_1

$\text{Enc}(\widetilde{A_4}, 1)$ $\Rightarrow$ Level 5

Product level: $12 = 2\kappa$

iO distinguishing attack

Reminder: iO

$$\forall C_1 \equiv C_2,\ O(C_1) \simeq_c O(C_2)$$

iO distinguishing attack

Reminder: iO

$$\forall C_1 \equiv C_2,\ O(C_1) \simeq_c O(C_2)$$

Objective: Find $C_1 \equiv C_2$ s.t. double mixed input product is 0 on C_1 and $\neq 0$ on C_2, e.g.

- the two mixed-input are $0 \mod p$ for C_1
  $\Rightarrow$ product is $0 \mod p^2$
- the two mixed-input are $\neq 0 \mod p$ for C_2
  $\Rightarrow$ product is $\neq 0 \mod p^2$

One example of C_1 and C_2

$$\boldsymbol{C_1}: \quad \begin{pmatrix}1 & 0\end{pmatrix} \quad \underset{x_1}{\begin{matrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\\ \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\end{matrix}} \quad \underset{x_2}{\begin{matrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\\ \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\end{matrix}} \quad \underset{x_1}{\begin{matrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\\ \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\end{matrix}} \quad \begin{pmatrix}0\\1\end{pmatrix} \qquad \Rightarrow \forall x,\ C_1(x) = 0$$

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$$\boldsymbol{C_2}: \quad \begin{pmatrix}1 & 0\end{pmatrix} \quad \underset{x_1}{\begin{matrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\\ \color{red}{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}}\end{matrix}} \quad \underset{x_2}{\begin{matrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\\ \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\end{matrix}} \quad \underset{x_1}{\begin{matrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\\ \color{red}{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}}\end{matrix}} \quad \begin{pmatrix}0\\ 1\end{pmatrix} \quad \Rightarrow \forall x,\ C_2(x) = 0$$

One example of C_1 and C_2

$$C_1: \quad \begin{pmatrix}1 & 0\end{pmatrix} \quad \underset{x_1}{\begin{matrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\\ \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\end{matrix}} \quad \underset{x_2}{\begin{matrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\\ \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\end{matrix}} \quad \underset{x_1}{\begin{matrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\\ \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\end{matrix}} \quad \begin{pmatrix}0\\1\end{pmatrix} \quad \Rightarrow \forall x,\ C_1(x) = 0$$

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- $C_1 \equiv C_2$

One example of C_1 and C_2

$$C_1: \quad \begin{pmatrix}1 & 0\end{pmatrix} \quad \underset{x_1}{\begin{pmatrix}1&0\\0&1\end{pmatrix} \atop \begin{pmatrix}1&0\\0&1\end{pmatrix}} \quad \underset{x_2}{\begin{pmatrix}1&0\\0&1\end{pmatrix} \atop \begin{pmatrix}1&0\\0&1\end{pmatrix}} \quad \underset{x_1}{\begin{pmatrix}1&0\\0&1\end{pmatrix} \atop \begin{pmatrix}1&0\\0&1\end{pmatrix}} \quad \begin{pmatrix}0\\1\end{pmatrix} \quad \Rightarrow \forall x,\ C_1(x) = 0$$

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- $C_1 \equiv C_2$
- the two mixed-input products are 0 for C_1

One example of C_1 and C_2

$$C_1: \quad \begin{pmatrix}1 & 0\end{pmatrix} \quad \begin{matrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} & \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} & \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\\ \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} & \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} & \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\\ x_1 & x_2 & x_1\end{matrix} \quad \begin{pmatrix}0\\1\end{pmatrix} \quad \Rightarrow \forall x,\ C_1(x) = 0$$

$$C_2: \quad \begin{pmatrix}1 & 0\end{pmatrix} \quad \begin{matrix}\color{blue}{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}} & \color{blue}{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}} & \color{red}{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}}\\ \color{red}{\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}} & \color{red}{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}} & \color{blue}{\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}}\\ x_1 & x_2 & x_1\end{matrix} \quad \begin{pmatrix}0\\1\end{pmatrix} \quad \Rightarrow \forall x,\ C_2(x) = 0$$

- $C_1 \equiv C_2$
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One example of C_1 and C_2

$$C_1: \quad \begin{pmatrix} 1 & 0 \end{pmatrix} \quad \underset{x_1}{\begin{matrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}} \quad \underset{x_2}{\begin{matrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}} \quad \underset{x_1}{\begin{matrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \Rightarrow \forall x, \ C_1(x) = 0$$

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- $C_1 \equiv C_2$
- the two mixed-input products are 0 for C_1
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> We can distinguish $O(C_1)$ from $O(C_2)$

Conclusion (1/2)

Counter-intuitive remark

This attack works only against the recent schemes
(with stronger security proofs)

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Why?

- Previous schemes prevent mixed-input attack using the randomization phase
  - difficult to get a security proof

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- New schemes use the mmap
  - easy to get a proof (in idealized model)

Conclusion (1/2)

Counter-intuitive remark

This attack works only against the recent schemes
(with stronger security proofs)

Why?

- Previous schemes prevent mixed-input attack using the randomization phase
  - difficult to get a security proof
- New schemes use the mmap
  - easy to get a proof (in idealized model)
- GGH13 mmap is not ideal
  - easier for an attacker to exploit its weakness

Conclusion (2/2)

Remarks

- Quantum poly time or classical $2^{O(\sqrt{n})}$ time

Conclusion (2/2)

Remarks

- Quantum poly time or classical $2^{O(\sqrt{n})}$ time
- Double mixed input attacks can be extended to circuit obfuscators

Conclusion (2/2)

iO (using GGH13) \ Attacks	Branching program obfuscators				Circuit obfuscators
	[GGH+13b]	[BR14]	[AGIS14, MSW14] [PST14, BGK+14] [BMSZ16]	[GMM+16]	[Zim15, AB15] [DGG+16]
[MSZ16]		✓	✓		
[CGH17]*	✓				
[CHKL18]†	✓	✓	✓	✓	
This talk‡			✓	✓	✓

* for input-partitionable branching programs

† for specific choices of parameters

‡ in the quantum setting

Conclusion (2/2)

Remarks

- Quantum poly time or classical $2^{O(\sqrt{n})}$ time
- Double mixed input attacks can be extended to circuit obfuscators
- [GGH$^+$13b]: only BP/circuit obfuscator currently standing in quantum

[GGH$^+$13b] S. Garg, C. Gentry, S. Halevi, M. Raykova, A. Sahai and B. Waters. Candidate indistinguishability obfuscation and functional encryption for all circuits, FOCS.

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Remarks

- Quantum poly time or classical $2^{O(\sqrt{n})}$ time
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Open problems

- Quantum attack against [GGH$^+$13b]

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- Quantum poly time or classical $2^{O(\sqrt{n})}$ time
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The GGH13 multilinear map

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- Sample g a "small" element in R.
  $\Rightarrow$ the plaintext space is $\mathcal{P} = R/\langle g \rangle$.
- Sample q a "large" integer.
  $\Rightarrow$ the encoding space is $R_q = R/(qR) = \mathbb{Z}_q[X]/(X^n + 1)$.

Notation

We write $[r]_q$ or $[r]$ the elements in R_q.

The GGH13 multilinear map: encodings

- Sample z uniformly in R_q.
- **Encoding:** An encoding of a at level i is

$$u = \left[\frac{a + rg}{z^i}\right]_q$$

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Addition and multiplication

Addition:

$$\left[\frac{a_1 + r_1 g}{z^i} \right]_q + \left[\frac{a_2 + r_2 g}{z^i} \right]_q = \left[\frac{a_1 + a_2 + r' g}{z^i} \right]_q .$$

Multiplication:

$$\left[\frac{a_1 + r_1 g}{z^i} \right]_q \cdot \left[\frac{a_2 + r_2 g}{z^j} \right]_q = \left[\frac{a_1 \cdot a_2 + r' g}{z^{i+j}} \right]_q .$$

The GGH13 multilinear map: zero-test

- Sample h in R of the order of $q^{1/2}$.
- Define

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Zero-test

To test if $u = [c/z^\kappa]$ is an encoding of zero (i.e. $c = 0 \mod g$), compute

$$[u \cdot p_{zt}]_q = [chg^{-1}]_q.$$

This is small iff c is a small multiple of g.

Quantum double-zero-test

Reminder

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$$[u p'_{zt}]_q \text{ small } \Leftrightarrow u = [cg^2/z^{2\kappa}]_q \text{ for some small } c$$
$$\Leftrightarrow u \text{ is a double zero at level } 2\kappa$$

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