# Theoretical obfuscation

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LIP, ENS de Lyon

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# Obfuscation

An obfuscator should:

- render the code of a program unintelligible;
- while preserving functionality and efficiency.

# Overview of the talk

1. Definition
2. Candidates
   - Security
   - Practicability
3. Example of construction of an obfuscator

# Outline of the talk

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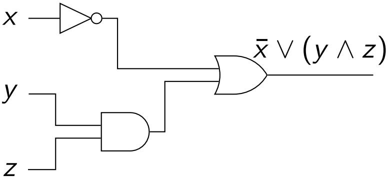


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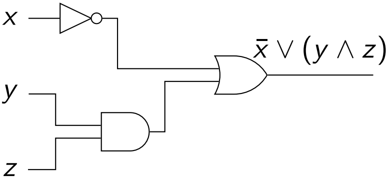
- Turing machine;
- Boolean circuit;
- Branching programs;

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VBB obfuscation is impossible to achieve [BGI$^+$01]

[BGI+01] B. Barak, O. Goldreich, R. Impagliazzo, S. Rudich, A. Sahai, S. Vadhan and K. Yang. On the (im) possibility of obfuscating programs, Crypto.

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Informally: anything revealed by $\mathcal{O}(C)$ is revealed by any $C' \equiv C$

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$C_1 \equiv C_2 \Rightarrow C = \mathcal{O}(C_1) \simeq_c \mathcal{O}(C_2)$ does not reveal $sk_1$ or $sk_2$

# Outline of the talk

# Disclaimer

We only have candidate iO

(no construction based on standard cryptographic assumptions)

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- Obfuscation via functional encryption
  - try to find the weakest primitive implying iO
  - some attacks and impossibility results (not well understood yet)
  - most of them are not instantiable

# Security

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All the candidate multilinear maps we know suffer from weaknesses.

| | number of candidates | still standing classically | still standing quantumly |
|---|---|---|---|
| Branching program iO | $\approx 20$ | $\approx 10$ | 3 |
| Circuit iO | $\approx 8$ | $\approx 8$ | 0 |

All attacks rely on the underlying multilinear map

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VBB obfuscators based on RLWE

# Practicability

| function obfuscated | security parameter $\lambda$ | size obfuscated program | obfuscation time | evaluation time | security assumption | reference |
|---|---|---|---|---|---|---|
| AES | 128 | 18 700 TB | | $10^{10}$ mults of $10^8$ bits integers | none - | [YLX17] |
| one-round key-exchange with 4 users | 52 | 4.8 GB | 2h20 | $\leq$ 1 min | none - | [CP18] |
| $A_1^{x_1} \times \cdots \times A_{20}^{x_{20}}$ | 80 | | 80 h | 25 min | none | [HHSSD17] |
| $x_1 \wedge \bar{x}_4 \wedge \cdots \wedge x_{32}$ | 53 | | 6.2 min | 32ms | entropic RLWE | [CDCG+18] |
| $x_1 \wedge \bar{x}_4 \wedge \cdots \wedge x_{64}$ | 73 | | 6.7h | 2.4s | entropic RLWE | [CDCG+18] |

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- two vectors $A_0$ and $A_{\ell+1}$,
- a function $\text{inp} : \{1, \dots, \ell\} \to \{1, \dots, r\}$ (where $r$ is the size of the input).

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\text{inp}(i)$ | 1 | 1 | 2 | 1 | 3 | 2 |

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$$A_0 \times \begin{matrix} A_{1,1} \\ A_{1,0} \end{matrix} \times \begin{matrix} A_{2,1} \\ A_{2,0} \end{matrix} \times \begin{matrix} A_{3,1} \\ A_{3,0} \end{matrix} \times \begin{matrix} A_{4,1} \\ A_{4,0} \end{matrix} \times \begin{matrix} A_{5,1} \\ A_{5,0} \end{matrix} \times \begin{matrix} A_{6,1} \\ A_{6,0} \end{matrix} \times A_7$$

# Branching programs

A branching program is a way of representing a function (like a Turing machine, or a circuit).

## A Branching Program (BP) is a collection of

- $2\ell$ matrices $A_{i,b}$ (for $i \in \{1, \dots, \ell\}$ and $b \in \{0, 1\}$),
- two vectors $A_0$ and $A_{\ell+1}$,
- a function inp : $\{1, \dots, \ell\} \to \{1, \dots, r\}$ (where $r$ is the size of the input).

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| inp($i$) | 1 | 1 | 2 | 1 | 3 | 2 |

$$x = 0 \quad 1 \quad 1$$

$$A_0 \times \begin{matrix} A_{1,1} \\ A_{1,0} \end{matrix} \times \begin{matrix} A_{2,1} \\ A_{2,0} \end{matrix} \times \begin{matrix} A_{3,1} \\ A_{3,0} \end{matrix} \times \begin{matrix} A_{4,1} \\ A_{4,0} \end{matrix} \times \begin{matrix} A_{5,1} \\ A_{5,0} \end{matrix} \times \begin{matrix} A_{6,1} \\ A_{6,0} \end{matrix} \times A_7 \begin{matrix} = 0 \to 0 \\ \neq 0 \to 1 \end{matrix}$$

# Cryptographic multilinear maps

## Definition: $\kappa$-multilinear map

Different levels of encodings, from 1 to $\kappa$.
Denote by $\mathrm{Enc}(a, i)$ a level-$i$ encoding of the message $a$.

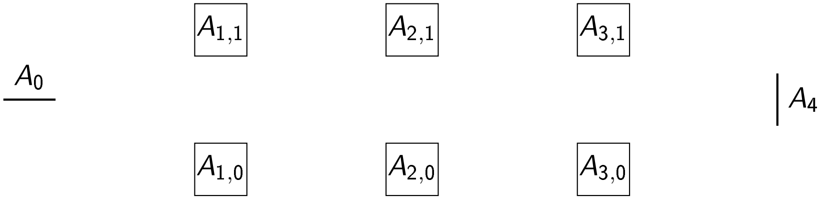
**Addition:** $\mathrm{Add}(\mathrm{Enc}(a_1, i), \mathrm{Enc}(a_2, i)) = \mathrm{Enc}(a_1 + a_2, i)$.

**Multiplication:** $\mathrm{Mult}(\mathrm{Enc}(a_1, i), \mathrm{Enc}(a_2, j)) = \mathrm{Enc}(a_1 \cdot a_2, i + j)$.

**Zero-test:** $\text{Zero-test}(\mathrm{Enc}(a, \kappa)) = \text{True iff } a = 0$.

# Simple obfuscator

- **Input:** A branching program
- Randomize the branching program
  - Add random diagonal blocks
  - Killian's randomization
  - Multiply by random (non zero) bundling scalars
- Encode the matrices using GGH13
- **Output:** The encoded matrices and vectors



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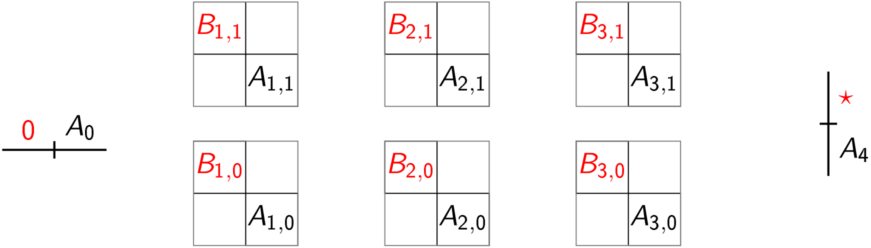

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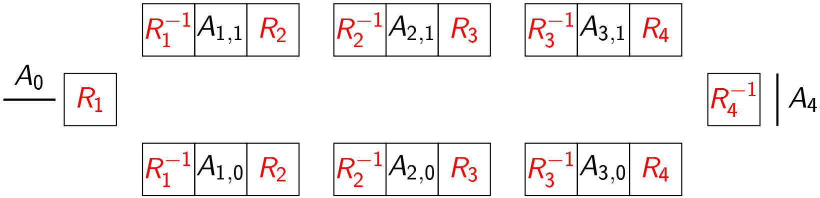

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$$\alpha_{1,1} \times \boxed{A_{1,1}} \quad \alpha_{2,1} \times \boxed{A_{2,1}} \quad \alpha_{3,1} \times \boxed{A_{3,1}}$$

$$\underline{A_0} \qquad\qquad\qquad\qquad\qquad\qquad \big| A_4$$

$$\alpha_{1,0} \times \boxed{A_{1,0}} \quad \alpha_{2,0} \times \boxed{A_{2,0}} \quad \alpha_{3,0} \times \boxed{A_{3,0}}$$

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$\widetilde{A_{1,1}}$ $\widetilde{A_{2,1}}$ $\widetilde{A_{3,1}}$

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$\mathrm{Enc}(\widetilde{A_{1,1}})$ $\mathrm{Enc}(\widetilde{A_{2,1}})$ $\mathrm{Enc}(\widetilde{A_{3,1}})$

$\mathrm{Enc}(\widetilde{A_0})$ $\mathrm{Enc}(\widetilde{A_4})$

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# Mixed-input attack

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- $A_{i,b}$ input branching program
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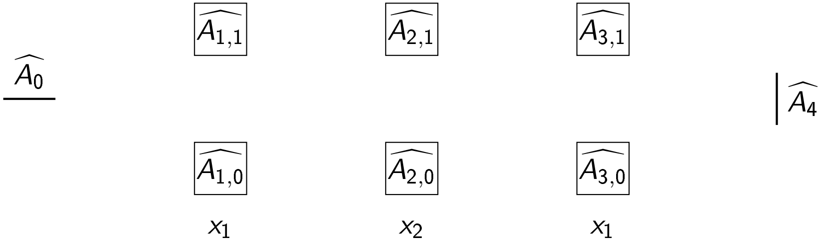

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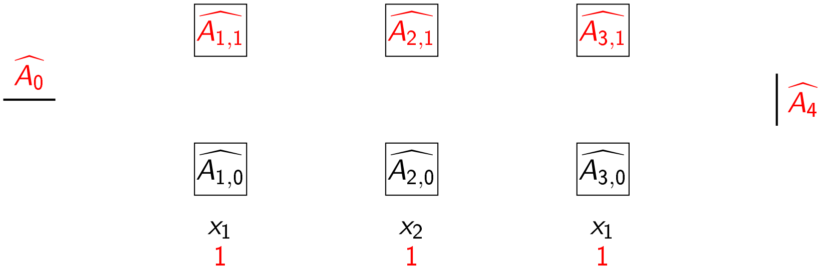

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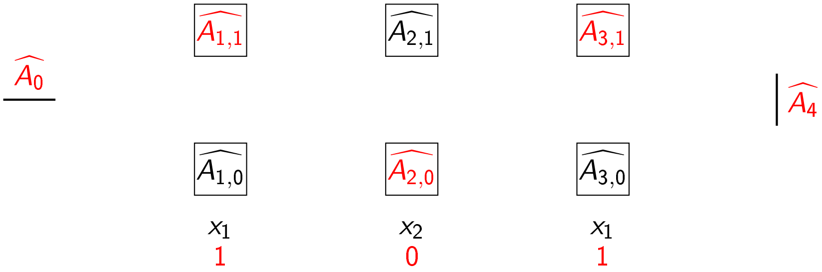

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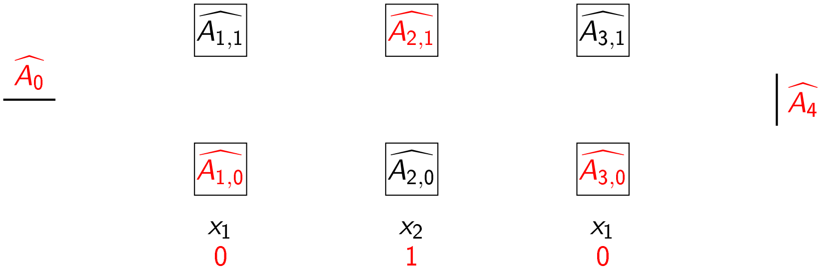

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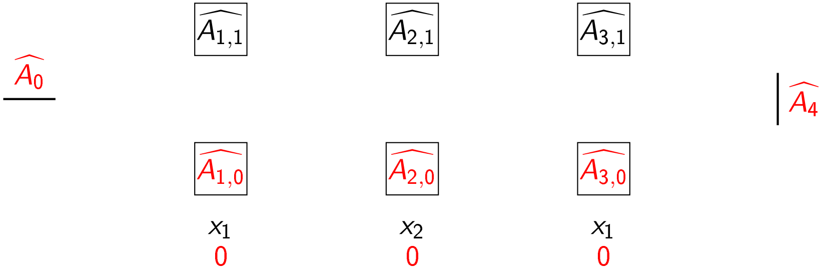

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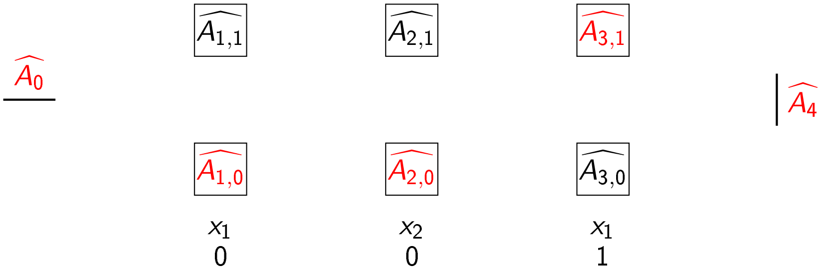

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$$\mathrm{Enc}(\widetilde{A_0}, 1) \quad \begin{matrix} \mathrm{Enc}(\boxed{\widetilde{A_{1,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,1}}}, 1) \\ \mathrm{Enc}(\boxed{\widetilde{A_{1,0}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,0}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,0}}}, 1) \\ x_1 & x_2 & x_1 \\ 0 & 0 & 1 \end{matrix} \quad \Big| \, \mathrm{Enc}(\widetilde{A_4}, 1)$$

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- In the randomization phase $\Rightarrow$ not in this talk
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$$x_1 \qquad x_2 \qquad x_1$$

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| $x_1$ | $x_2$ | $x_1$ |
|---|---|---|
| 0 | 0 | 1 |

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| $x_1$ | $x_2$ | $x_1$ |
|---|---|---|
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Total level: 7 ⇒ cannot zero-test

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Questions?

# References I


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