Program obfuscation

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Overview of the talk

1. Definition
2. Candidates
   - Security
   - Practicability
3. Example of construction of an obfuscator

Outline of the talk

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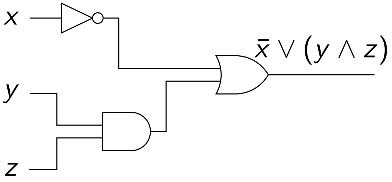

Notation

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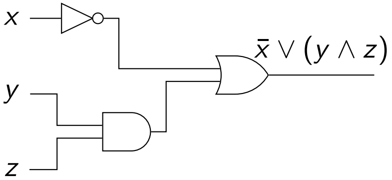
- C/C++/Python/$\cdots$ code;
- Turing machine;
- Boolean circuit;
- Branching programs;



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VBB obfuscation is impossible to achieve [BGI+01]

[BGI+01] B. Barak, O. Goldreich, R. Impagliazzo, S. Rudich, A. Sahai, S. Vadhan and K. Yang. On the (im) possibility of obfuscating programs, Crypto.

Indistinguishability Obfuscation (iO)

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- (indistinguishability) For all $C_1, C_2 \in \mathcal{C}$ with $C_1 \equiv C_2$,

$$\mathcal{O}(C_1) \simeq_c \mathcal{O}(C_2).$$

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$$\mathcal{O}(C) = \text{smallest circuit computing the same function as } C$$

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Informally: anything revealed by $\mathcal{O}(C)$ is revealed by any $C' \equiv C$

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  - $sk_1 \leftarrow \texttt{Setup}()$, $sk_2 \leftarrow \texttt{Setup}()$
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$C_1 \equiv C_2 \Rightarrow C = \mathcal{O}(C_1) \simeq_c \mathcal{O}(C_2)$ does not reveal sk_1 or sk_2

Outline of the talk

Disclaimer

We only have candidate iO

(no construction based on standard cryptographic assumptions)

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- Obfuscation via functional encryption
  - try to find the weakest primitive implying iO
  - some attacks and impossibility results (not well understood yet)
  - most of them are not instantiable

Security

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	number of candidates	still standing classically	still standing quantumly
Branching program iO	≈ 20	≈ 10	3
Circuit iO	≈ 8	≈ 8	0

All attacks rely on the underlying multilinear map

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- compute-and-compare functions

$$f_{g,y}(x) = 1 \text{ iff } g(x) = y$$

Practicability

function obfuscated	security parameter λ	size obfuscated program	obfuscation time	evaluation time	security assumption	reference
AES	128	18 700 TB		10^{10} mults of 10^8 bits integers	none -	[YLX17]
one-round key-exchange with 4 users	52	4.8 GB	2h20	≤ 1 min	none -	[CP18]
$A_1^{x_1} \times \cdots \times A_{20}^{x_{20}}$	80		80 h	25 min	none	[HHSSD17]
$x_1 \wedge \bar{x}_4 \wedge \cdots \wedge x_{32}$	53		6.2 min	32ms	entropic RLWE	[CDCG+18]
$x_1 \wedge \bar{x}_4 \wedge \cdots \wedge x_{64}$	73		6.7h	2.4s	entropic RLWE	[CDCG+18]

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- 2ℓ matrices $M_{i,b}$ (for $i \in \{1, \dots, \ell\}$ and $b \in \{0, 1\}$),
- two vectors M_0 and $M_{\ell+1}$,
- a vector inp $\in \{1, \dots, r\}^\ell$ (where r is the size of the input).

BP

	x_1	x_1	x_2	x_1	x_3	x_2	
M_0	$M_{1,1}$	$M_{2,1}$	$M_{3,1}$	$M_{4,1}$	$M_{5,1}$	$M_{6,1}$	M_7
	$M_{1,0}$	$M_{2,0}$	$M_{3,0}$	$M_{4,0}$	$M_{5,0}$	$M_{6,0}$	

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BP

$$M_0 \times \begin{array}{c} x_1 \\ M_{1,1} \\ M_{1,0} \end{array} \times \begin{array}{c} x_1 \\ M_{2,1} \\ M_{2,0} \end{array} \times \begin{array}{c} x_2 \\ M_{3,1} \\ M_{3,0} \end{array} \times \begin{array}{c} x_1 \\ M_{4,1} \\ M_{4,0} \end{array} \times \begin{array}{c} x_3 \\ M_{5,1} \\ M_{5,0} \end{array} \times \begin{array}{c} x_2 \\ M_{6,1} \\ M_{6,0} \end{array} \times M_7$$

Evaluation on $x = 0 \quad 1 \quad 1$

Branching programs

A branching program represents a function (cf Turing machine, or circuit).

A Branching Program (BP) is a collection of

- 2ℓ matrices $M_{i,b}$ (for $i \in \{1,\ldots,\ell\}$ and $b \in \{0,1\}$),
- two vectors M_0 and $M_{\ell+1}$,
- a vector inp $\in \{1,\ldots,r\}^\ell$ (where r is the size of the input).

BP

$$
\begin{array}{ccccccccccccccc}
 & & x_1 & & x_1 & & x_2 & & x_1 & & x_3 & & x_2 & & \\
M_0 & \times & \begin{array}{c} M_{1,1} \\ M_{1,0} \end{array} & \times & \begin{array}{c} M_{2,1} \\ M_{2,0} \end{array} & \times & \begin{array}{c} M_{3,1} \\ M_{3,0} \end{array} & \times & \begin{array}{c} M_{4,1} \\ M_{4,0} \end{array} & \times & \begin{array}{c} M_{5,1} \\ M_{5,0} \end{array} & \times & \begin{array}{c} M_{6,1} \\ M_{6,0} \end{array} & \times & M_7 \begin{array}{l} = 0 \to 0 \\ \neq 0 \to 1 \end{array}
\end{array}
$$

Evaluation on $x = 0 \quad 1 \quad 1$

Cryptographic multilinear maps

Definition: κ-multilinear map

Different levels of encodings, from 1 to κ.
Write $\text{Enc}(a, i)$ a level-i encoding of the message a.

Addition: $\text{Add}(\text{Enc}(a_1, i), \text{Enc}(a_2, i)) = \text{Enc}(a_1 + a_2, i)$.

Multiplication: $\text{Mult}(\text{Enc}(a_1, i), \text{Enc}(a_2, j)) = \text{Enc}(a_1 \cdot a_2, i + j)$.

Zero-test: $\text{Zero-test}(\text{Enc}(a, \kappa)) = \text{True iff } a = 0$.

Simple obfuscator

[GGH$^+$13, BR14, BGK$^+$14, PST14, AGIS14, MSW14, GMM$^+$16]

- **Input:** A branching program
- Randomize the branching program
  - Add random diagonal blocks
  - Killian's randomization
  - Multiply by random (non zero) bundling scalars
- Encode the matrices using a multilinear map
- **Output:** The encoded matrices and vectors

$$\boxed{A_{1,1}} \quad \boxed{A_{2,1}} \quad \boxed{A_{3,1}}$$

$$\underline{A_0} \qquad\qquad\qquad\qquad\qquad\qquad \Big| A_4$$

$$\boxed{A_{1,0}} \quad \boxed{A_{2,0}} \quad \boxed{A_{3,0}}$$

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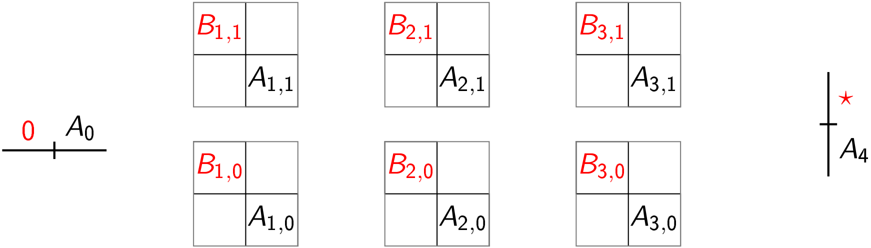

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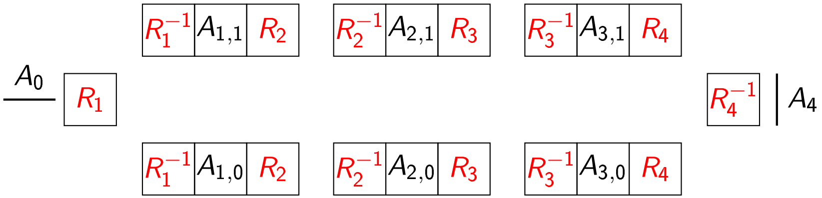

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$$\alpha_{1,1} \times \boxed{A_{1,1}} \qquad \alpha_{2,1} \times \boxed{A_{2,1}} \qquad \alpha_{3,1} \times \boxed{A_{3,1}}$$

$$\underline{A_0} \qquad\qquad\qquad\qquad\qquad\qquad \Big| A_4$$

$$\alpha_{1,0} \times \boxed{A_{1,0}} \qquad \alpha_{2,0} \times \boxed{A_{2,0}} \qquad \alpha_{3,0} \times \boxed{A_{3,0}}$$

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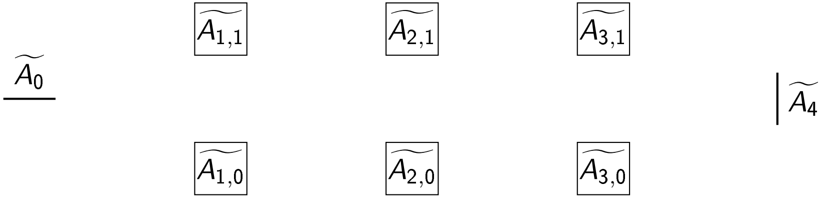

Simple obfuscator

[GGH+13, BR14, BGK+14, PST14, AGIS14, MSW14, GMM+16]

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- Encode the matrices using a multilinear map
- **Output:** The encoded matrices and vectors

$$\mathrm{Enc}(\widetilde{A_0}) \quad \begin{array}{ccc} \mathrm{Enc}\left(\boxed{\widetilde{A_{1,1}}}\right) & \mathrm{Enc}\left(\boxed{\widetilde{A_{2,1}}}\right) & \mathrm{Enc}\left(\boxed{\widetilde{A_{3,1}}}\right) \\ \\ \mathrm{Enc}\left(\boxed{\widetilde{A_{1,0}}}\right) & \mathrm{Enc}\left(\boxed{\widetilde{A_{2,0}}}\right) & \mathrm{Enc}\left(\boxed{\widetilde{A_{3,0}}}\right) \end{array} \quad \mathrm{Enc}(\widetilde{A_4})$$

Mixed-input attack

Notations

- $A_{i,b}$ input branching program
- $\widetilde{A_{i,b}}$ after randomisation
- $\widehat{A_{i,b}}$ after encoding with a multilinear map (output of the iO)

$$
\begin{array}{ccccc}
 & \boxed{\widehat{A_{1,1}}} & \boxed{\widehat{A_{2,1}}} & \boxed{\widehat{A_{3,1}}} & \\
\underline{\widehat{A_0}} & & & & \Big|\,\widehat{A_4} \\
 & \boxed{\widehat{A_{1,0}}} & \boxed{\widehat{A_{2,0}}} & \boxed{\widehat{A_{3,0}}} & \\
 & x_1 & x_2 & x_1 &
\end{array}
$$

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Mixed-input attack

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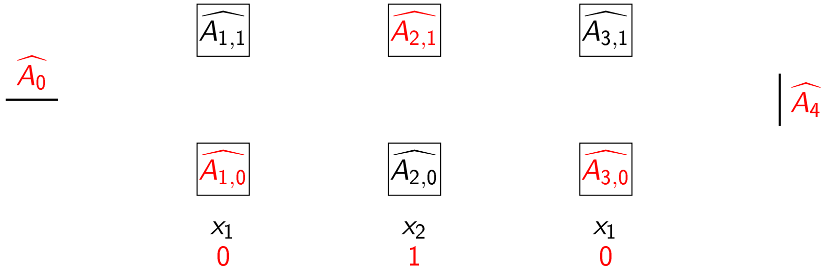

Mixed-input attack

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- $\widehat{A_{i,b}}$ after encoding with a multilinear map (output of the iO)

$$\underline{\widehat{A_0}} \qquad \begin{matrix} \boxed{\widehat{A_{1,1}}} & \boxed{\widehat{A_{2,1}}} & \boxed{\widehat{A_{3,1}}} \\ \boxed{\widehat{A_{1,0}}} & \boxed{\widehat{A_{2,0}}} & \boxed{\widehat{A_{3,0}}} \\ x_1 & x_2 & x_1 \\ 0 & 0 & 0 \end{matrix} \qquad \Big| \, \widehat{A_4}$$

Mixed-input attack

Notations

- $A_{i,b}$ input branching program
- $\widetilde{A_{i,b}}$ after randomisation
- $\widehat{A_{i,b}}$ after encoding with a multilinear map (output of the iO)

	$\boxed{\widehat{A_{1,1}}}$	$\boxed{\widehat{A_{2,1}}}$	$\boxed{\widehat{A_{3,1}}}$	
$\widehat{A_0}$				$\widehat{A_4}$
	$\boxed{\widehat{A_{1,0}}}$	$\boxed{\widehat{A_{2,0}}}$	$\boxed{\widehat{A_{3,0}}}$	
	x_1	x_2	x_1	
	0	0	1	

Mixed-input attack

Notations

- $A_{i,b}$ input branching program
- $\widetilde{A_{i,b}}$ after randomisation
- $\widehat{A_{i,b}}$ after encoding with a multilinear map (output of the iO)

$$
\begin{array}{ccccc}
 & \mathrm{Enc}(\boxed{\widetilde{A_{1,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,1}}}, 1) & \\
\underline{\mathrm{Enc}(\widetilde{A_0}, 1)} & & & & \Big| \, \mathrm{Enc}(\widetilde{A_4}, 1) \\
 & \mathrm{Enc}(\boxed{\widetilde{A_{1,0}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,0}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,0}}}, 1) & \\
 & x_1 & x_2 & x_1 & \\
 & 0 & 0 & 1 &
\end{array}
$$

Preventing mixed-input attacks

- In the randomization phase $\Rightarrow$ not in this talk [GGH+13, BR14]
- Using the mmap $\Rightarrow$ straddling set system
  [BGK+14, PST14, AGIS14, MSW14, GMM+16]

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Mmap degree: $\kappa = 5$

$$\begin{array}{ccccc} & \mathrm{Enc}(\boxed{\widetilde{A_{1,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,1}}}, 1) & \\ \underline{\mathrm{Enc}(\widetilde{A_0}, 1)} & & & & \Big| \mathrm{Enc}(\widetilde{A_4}, 1) \\ & \mathrm{Enc}(\boxed{\widetilde{A_{1,0}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,0}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,0}}}, 1) & \\ & x_1 & x_2 & x_1 & \end{array}$$

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Mmap degree: $\kappa = 6$

$$\mathrm{Enc}(\widetilde{A_0}, 1) \quad \begin{matrix} \mathrm{Enc}(\boxed{\widetilde{A_{1,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,1}}}, 2) \\ \\ \mathrm{Enc}(\boxed{\widetilde{A_{1,0}}}, 2) & \mathrm{Enc}(\boxed{\widetilde{A_{2,0}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,0}}}, 1) \\ x_1 & x_2 & x_1 \end{matrix} \quad \Big| \, \mathrm{Enc}(\widetilde{A_4}, 1)$$

Preventing mixed-input attacks

- In the randomization phase $\Rightarrow$ not in this talk [GGH$^+$13, BR14]
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$$\underline{\mathrm{Enc}(\widetilde{A_0}, 1)} \quad \begin{array}{ccc} \mathrm{Enc}(\boxed{\widetilde{A_{1,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{2,1}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,1}}}, 2) \\ \\ \mathrm{Enc}(\boxed{\widetilde{A_{1,0}}}, 2) & \mathrm{Enc}(\boxed{\widetilde{A_{2,0}}}, 1) & \mathrm{Enc}(\boxed{\widetilde{A_{3,0}}}, 1) \\ x_1 & x_2 & x_1 \\ 0 & 0 & 1 \end{array} \quad \Big| \, \mathrm{Enc}(\widetilde{A_4}, 1)$$

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Total level: 7 $\Rightarrow$ cannot zero-test

What to remember

+ iO would be very useful (at least for theory) ...

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+ iO would be very useful (at least for theory) . . .
− . . . but no constructions from standard assumptions yet

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+ maybe for restricted class of functions efficiency and security are possible

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Questions?

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History (GGH13-based branching program obfuscation)

	2013	2014	2015	2016	2017	2018
Constructions	[GGH$^+$13]	[BR14] [BGK$^+$14, PST14] [AGIS14, MSW14]		[GMM$^+$16]	[FRS17]	
Attacks				[MSZ16]	[CGH17] [ADGM17]	[CHKL18] [Pel18]

History (GGH13-based branching program obfuscation)

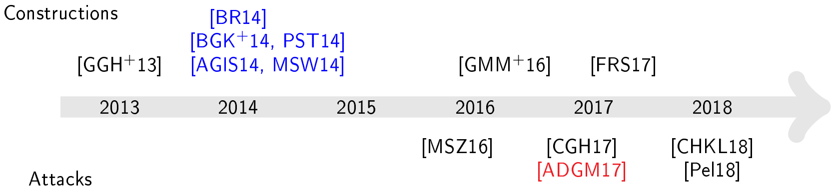
Constructions

	2013	2014	2015	2016	2017	2018
Constructions	[GGH+13]	[BR14] [BGK+14, PST14] [AGIS14, MSW14]		[GMM+16]	[FRS17]	
Attacks				[MSZ16]	[CGH17] [ADGM17]	[CHKL18] [Pel18]

Attacks

[MSZ16]: all constructions without diagonal blocks

History (GGH13-based branching program obfuscation)



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[ADGM17]: idem MSZ but from circuits

History (GGH13-based branching program obfuscation)

Constructions

[GGH$^+$13]

[BR14]
[BGK$^+$14, PST14]
[AGIS14, MSW14]

[GMM$^+$16]

[FRS17]

2013 2014 2015 2016 2017 2018

[MSZ16]

[CGH17]
[ADGM17]

[CHKL18]
[Pel18]

Attacks

[MSZ16]: all constructions without diagonal blocks

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[CGH17]: use input-partitionability (cf CLT13)

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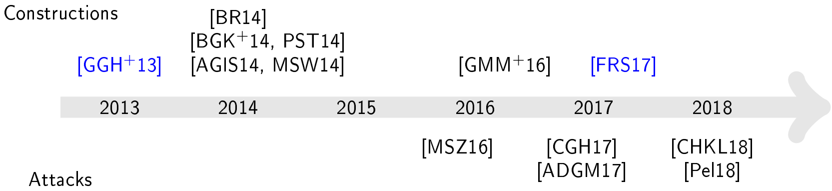

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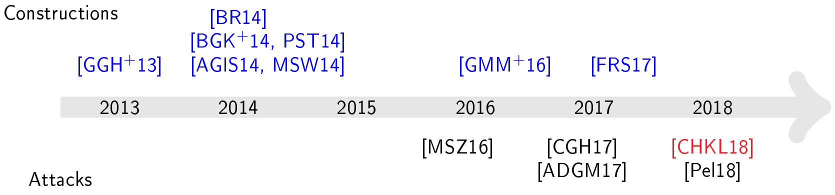

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[CHKL18]: NTRU attack for specific choices of parameters

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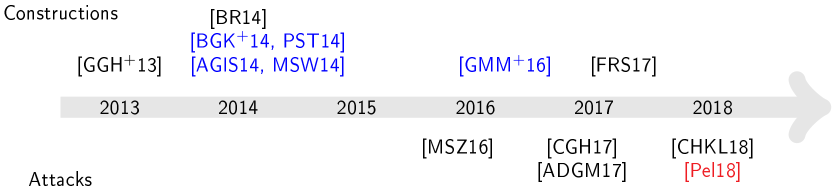

[MSZ16]: all constructions without diagonal blocks

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[CGH17]: use input-partitionability (cf CLT13) $\Rightarrow$ prevented by [FRS17]

[CHKL18]: NTRU attack for specific choices of parameters

[Pel18]: quantum attack

Current status

iOs / Attacks	[GGH$^+$13]	[BR14, BGK$^+$14, PST14, AGIS14, MSW14]	[GMM$^+$16]	circuit obfuscators [Zim15, AB15, DGG$^+$18]
[MSZ16]		fully broken		
[CGH17]	input-partitionable			
[CHKL18]	some parameters		some parameters	
[Pel18]			quantum	quantum

Still standing classically:

- [GGH$^+$13]+[FRS17]
- [GMM$^+$16]
- all circuit obfuscators

Still standing quantumly:

- [GGH$^+$13]+[FRS17]