Attacks on GGH13-based obfuscation

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ENS de Lyon

Opacity workshop
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Indistinguishability Obfuscation (iO)

$$C \equiv C'$$

$C(x) = C'(x)$ for all x

$$\Downarrow$$

$$\mathcal{O}(C) \approx_c \mathcal{O}(C')$$

3 categories

- Branching program obfuscation
- Circuit obfuscation
- Obfuscation from functional encryption

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Overview of the talk

1. Simple obfuscator
2. Quantum attack
3. State-of-the-art

Outline of the talk

1. Simple obfuscator
2. Quantum attack
3. State-of-the-art

Branching programs

A branching program represents a function (cf Turing machine, or circuit).

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A Branching Program (BP) is a collection of

- 2ℓ matrices $M_{i,b}$ (for $i \in \{1, \ldots, \ell\}$ and $b \in \{0, 1\}$),
- two vectors M_0 and $M_{\ell+1}$,
- a vector inp $\in \{1, \ldots, r\}^\ell$ (where r is the size of the input).

BP

	x_1	x_1	x_2	x_1	x_3	x_2	
M_0	$M_{1,1}$	$M_{2,1}$	$M_{3,1}$	$M_{4,1}$	$M_{5,1}$	$M_{6,1}$	M_7
	$M_{1,0}$	$M_{2,0}$	$M_{3,0}$	$M_{4,0}$	$M_{5,0}$	$M_{6,0}$	

Evaluation on $x = 0 \quad 1 \quad 1$

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$M_0 \times$	$M_{1,1}$ / $M_{1,0}$ $\times$	$M_{2,1}$ / $M_{2,0}$	$M_{3,1}$ / $M_{3,0}$	$M_{4,1}$ / $M_{4,0}$	$M_{5,1}$ / $M_{5,0}$	$M_{6,1}$ / $M_{6,0}$	M_7

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$$M_0 \times \begin{matrix} x_1 \\ M_{1,1} \\ M_{1,0} \end{matrix} \times \begin{matrix} x_1 \\ M_{2,1} \\ M_{2,0} \end{matrix} \times \begin{matrix} x_2 \\ M_{3,1} \\ M_{3,0} \end{matrix} \times \begin{matrix} x_1 \\ M_{4,1} \\ M_{4,0} \end{matrix} \times \begin{matrix} x_3 \\ M_{5,1} \\ M_{5,0} \end{matrix} \times \begin{matrix} x_2 \\ M_{6,1} \\ M_{6,0} \end{matrix} \quad M_7$$

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Cryptographic multilinear maps (asymmetric setting)

Different levels of encodings, parametrized by sets $S \subseteq \{1, \cdots, \kappa\}$.

Definition: asymmetric multilinear map

$\mathrm{Enc}(a, S)$: encoding of a at level S.
$S^* = \{1, \cdots, \kappa\}$, maximum level.

Addition: $\mathrm{Add}(\mathrm{Enc}(a_1, S), \mathrm{Enc}(a_2, S)) = \mathrm{Enc}(a_1 + a_2, S)$.

Multiplication: if $S_1 \cap S_2 = \emptyset$,

$$\mathrm{Mult}(\mathrm{Enc}(a_1, S_1), \mathrm{Enc}(a_2, S_2)) = \mathrm{Enc}(a_1 \cdot a_2, S_1 \cup S_2).$$

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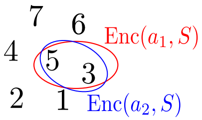

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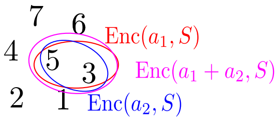

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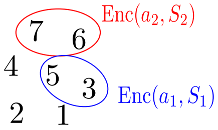

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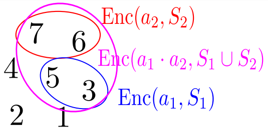

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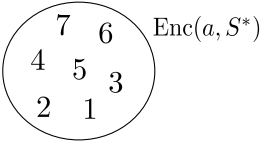
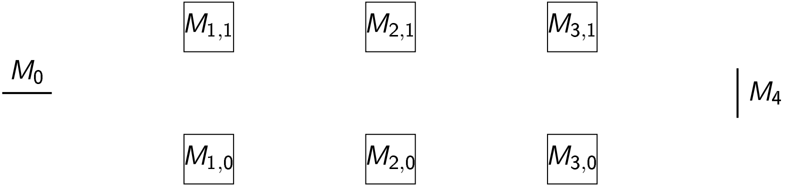


Simple obfuscator

- **Input:** A branching program
- Randomize the branching program
  - Add random diagonal blocks
  - Killian's randomization
  - Multiply by random (non zero) bundling scalars
- Encode the matrices using mmap
- **Output:** The encoded matrices and vectors



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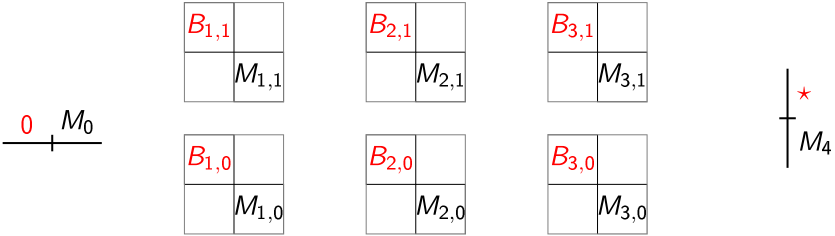

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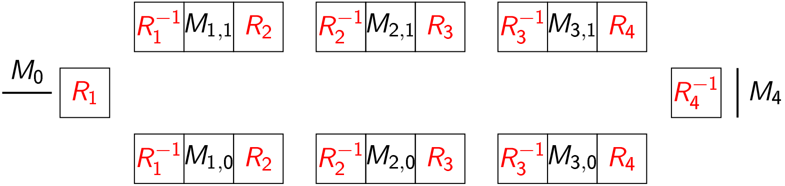

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$\alpha_{1,1} \times \boxed{M_{1,1}}$ $\alpha_{2,1} \times \boxed{M_{2,1}}$ $\alpha_{3,1} \times \boxed{M_{3,1}}$

$\underline{M_0}$ $\big| M_4$

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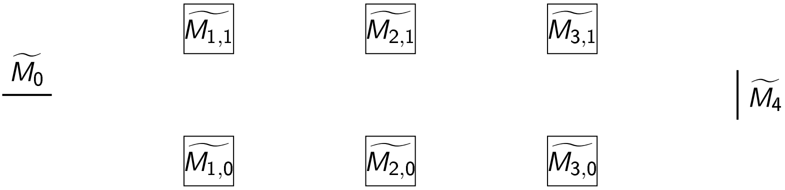

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$S^* = \{1, 2, 3, 4, 5\}$

$\text{Enc}(\widetilde{M_{1,1}}, \{2\})$ $\text{Enc}(\widetilde{M_{2,1}}, \{3\})$ $\text{Enc}(\widetilde{M_{3,1}}, \{4\})$

$\text{Enc}(\widetilde{M_0}, \{1\})$ $\text{Enc}(\widetilde{M_4}, \{5\})$

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Outline of the talk

1 Simple obfuscator

2 Quantum attack

3 State-of-the-art

GGH13 in a quantum world

Reminder: asymmetric multilinear map

$\mathrm{Enc}(a, S)$ = encoding of a at level S.
$S^* = \{1, \cdots, \kappa\}$ maximum level.

Addition: $\mathrm{Add}(\mathrm{Enc}(a_1, S), \mathrm{Enc}(a_2, S)) = \mathrm{Enc}(a_1 + a_2, S)$.

Multiplication: if $S_1 \cap S_2 = \emptyset$,

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GGH13 in a quantum world

The GGH13 map

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Zero-test: $\text{Zero-test}(\text{Enc}(a, S^*))$ = True iff $a = 0 \bmod p$.

e.g. $\{1, 3, 4\} \uplus \{2, 3\} = \{1, 2, 3, 3, 4\}$

GGH13 in a quantum world

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$2S^* = S^* \uplus S^* = \{1, 1, 2, 2, \cdots, \kappa, \kappa\}$

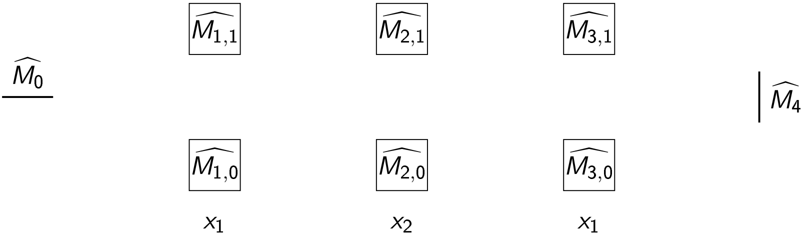
With a quantum computer

$$\text{Double-zero-test}(\text{Enc}(a, 2S^*)) = \text{True iff } a = 0 \text{ mod } p^2$$

Mixed-input attack

Notations

- $M_{i,b}$ input branching program
- $\widetilde{M_{i,b}}$ after randomisation
- $\widehat{M_{i,b}}$ after encoding with GGH13 map (output of the iO)



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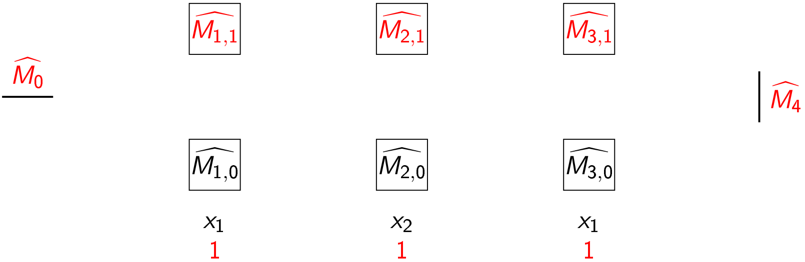

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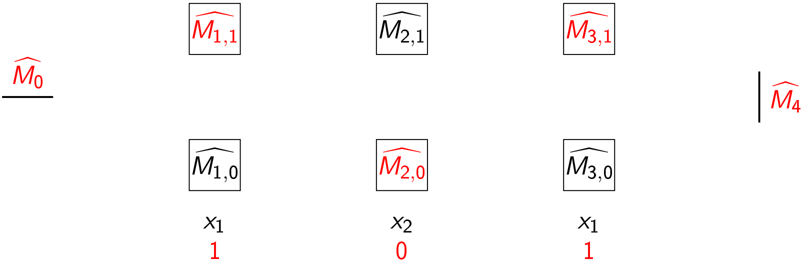

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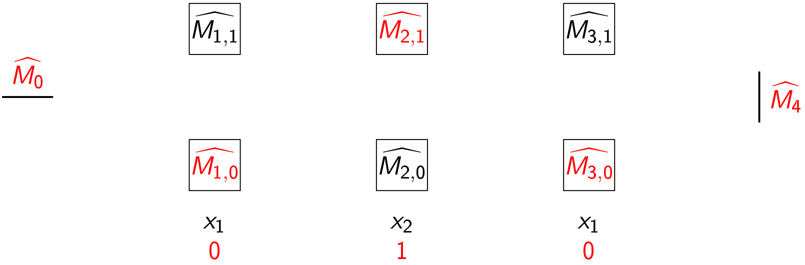

Mixed-input attack

Notations

- $M_{i,b}$ input branching program
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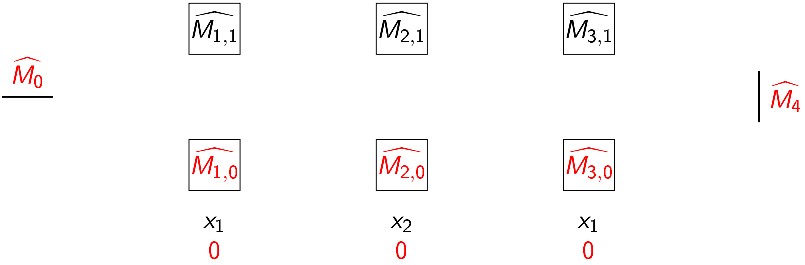

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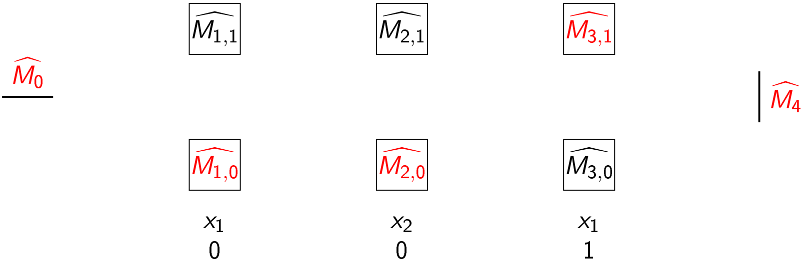

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	$\text{Enc}(\boxed{\widetilde{M_{1,1}}},\{2\})$	$\text{Enc}(\boxed{\widetilde{M_{2,1}}},\{3\})$	$\text{Enc}(\boxed{\widetilde{M_{3,1}}},\{4\})$	
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	x_1	x_2	x_1	
	0	0	1	

Preventing mixed-input attacks

- In the randomization phase $\Rightarrow$ not in this talk

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Total level: $\{1, 2, 3, 4, 5, 6, 6\} \Rightarrow$ cannot zero-test

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Generalisation: $\{1\}, \{2,3\}, \{4,5\}, \{6,7\}$
$\{1,2\}, \{3,4\}, \{5,6\}, \{7\}$

Attack idea: double mixed input

Reminder

In quantum world, we have

$$\text{Double-zero-test}(\text{Enc}(a, 2S^*)) = \text{True iff } a = 0 \mod p^2$$

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Product level: $S^* \uplus S^* = 2S^*$

iO distinguishing attack

Reminder: iO

$$\forall C_1 \equiv C_2,\ O(C_1) \simeq_c O(C_2)$$

iO distinguishing attack

Reminder: iO

$$\forall C_1 \equiv C_2,\ O(C_1) \simeq_c O(C_2)$$

Objective: Find $C_1 \equiv C_2$ s.t. double mixed input product is 0 on C_1 and $\neq 0$ on C_2, e.g.

- the two mixed-input are $0 \mod p$ for C_1
  $\Rightarrow$ product is $0 \mod p^2$
- the two mixed-input are $\neq 0 \mod p$ for C_2
  $\Rightarrow$ product is $\neq 0 \mod p^2$

One example of C_1 and C_2

$$\boldsymbol{C_1}: \quad \begin{pmatrix}1 & 0\end{pmatrix} \quad \begin{matrix} \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} & \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} & \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \\ \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} & \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} & \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \\ x_1 & x_2 & x_1 \end{matrix} \quad \begin{pmatrix}0\\1\end{pmatrix} \quad \Rightarrow \forall x,\ C_1(x) = 0$$

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One example of C_1 and C_2

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- $C_1 \equiv C_2$

One example of C_1 and C_2

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\mathbf{C_1}: \quad \begin{pmatrix} 1 & 0 \end{pmatrix} \quad
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\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
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$$

- $C_1 \equiv C_2$
- the two mixed-input products are 0 for C_1

One example of C_1 and C_2

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One example of C_1 and C_2

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- $C_1 \equiv C_2$
- the two mixed-input products are 0 for C_1
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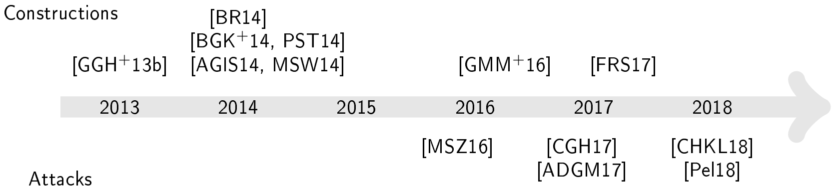
We can distinguish $\mathcal{O}(C_1)$ from $\mathcal{O}(C_2)$

Outline of the talk

History (GGH13-based branching program obfuscation)



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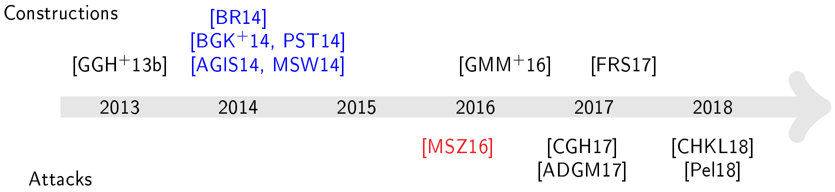

[MSZ16]: all constructions without diagonal blocks

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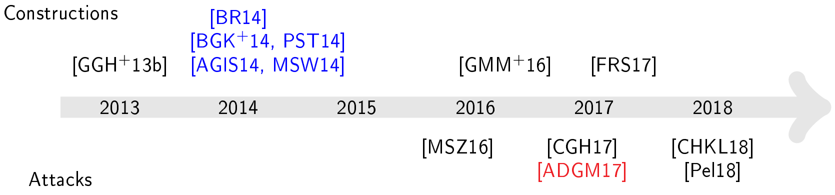

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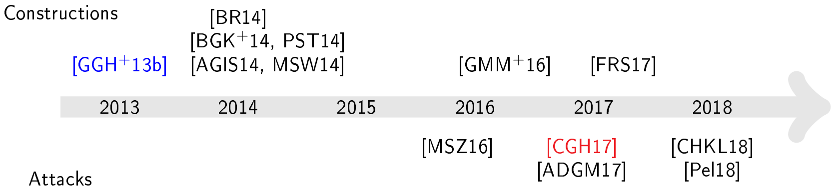

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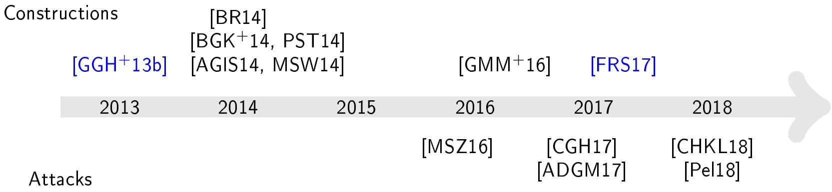

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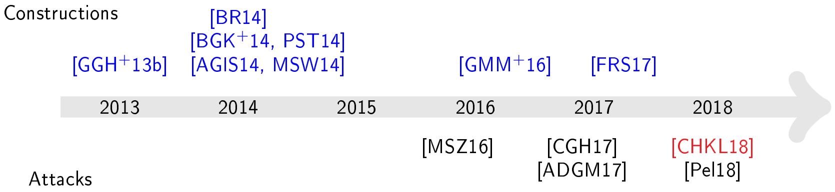

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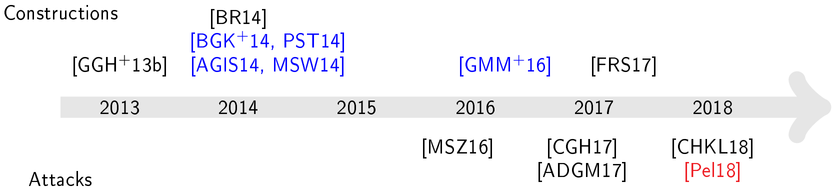

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Current status

Attacks \ iOs	[GGH+13b]	[BR14, BGK+14, PST14, AGIS14, MSW14]	[GMM+16]	circuit obfuscators [Zim15, AB15, DGG+18]
[MSZ16]		fully broken		
[CGH17]	input-partitionable			
[CHKL18]	some parameters		some parameters	
[Pel18]			quantum	quantum

Still standing classically:

- [GGH+13b]+[FRS17]
- [GMM+16]
- all circuit obfuscators

Still standing quantumly:

- [GGH+13b]+[FRS17]

Current status

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Still standing quantumly:

- [GGH+13b]+[FRS17]

Questions?

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