## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )

$$
M=\left(\begin{array}{cc}
10 & 7 \\
2 & 2
\end{array}\right)
$$

## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )



$$
M=\left(\begin{array}{cc}
10 & 7 \\
2 & 2
\end{array}\right)
$$

Compute QR factorization

## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )

$$
M=\left(\begin{array}{cc}
10.2 & 7.3 \\
0 & 0.6
\end{array}\right)
$$

## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )

$$
M=\left(\begin{array}{cc}
10.2 & 7.3 \\
0 & 0.6
\end{array}\right)
$$

"Euclidean division" (over $\mathbb{R}$ )

$$
\text { of } 7.3 \text { by } 10.2
$$

## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )

$$
M=\left(\begin{array}{cc}
10.2 & -2.9 \\
0 & 0.6
\end{array}\right)
$$

## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )

$$
M=\left(\begin{array}{cc}
-2.9 & 10.2 \\
0.6 & 0
\end{array}\right)
$$

swap

## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )

$$
M=\left(\begin{array}{cc}
-2.9 & 10.2 \\
0.6 & 0
\end{array}\right)
$$

start again

## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )



$$
M=\left(\begin{array}{cc}
-2.9 & 10.2 \\
0.6 & 0
\end{array}\right)
$$

rotation

## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )

$$
M=\left(\begin{array}{cc}
3 & -10 \\
0 & -2
\end{array}\right)
$$

rotation

## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )

$$
M=\left(\begin{array}{cc}
3 & -10 \\
0 & -2
\end{array}\right)
$$

reduce $b_{2}$ with $b_{1}$
"Euclidean division" (over $\mathbb{R}$ ) of -10 by 3

## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )

$$
M=\left(\begin{array}{ll}
3 & -1 \\
0 & -2
\end{array}\right)
$$

## Lagrange-Gauss algorithm (over $\mathbb{Z}$ )

$$
M=\left(\begin{array}{ll}
3 & -1 \\
0 & -2
\end{array}\right)
$$

For Lagrange-Gauss algorithm over $R$, we need

- rotation (i.e., QR factorization)
- Euclidean division

