Approx-SVP in Ideal lattices with Pre-Processing

Alice Pellet-Mary, Guillaume Hanrot and Damien Stehlé

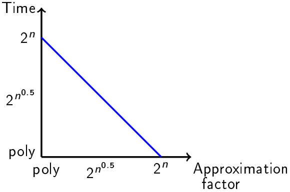
LIP, ENS de Lyon (France)

tMeet seminar at IIT Madras,
October 30, 2018

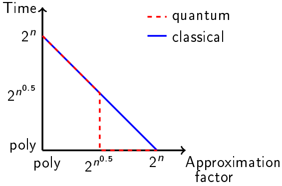
European Research Council
Established by the European Commission

What is this talk about

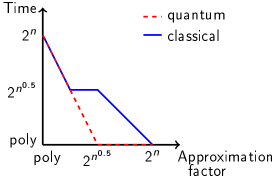
Time/Approximation factor trade-off for SVP in ideal lattices:



BKZ algorithm

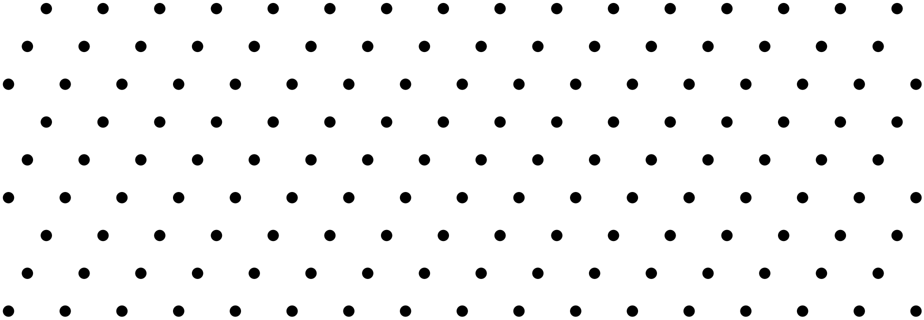


[CDPR16,CDW17]



This work
(with $2^{O(n)}$ pre-processing)

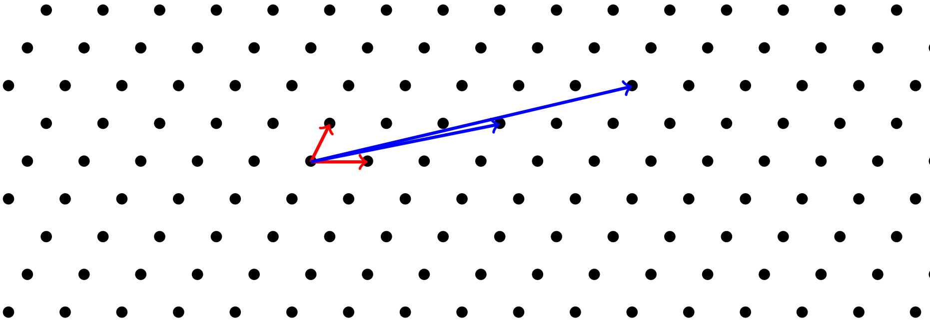
Lattices



Lattice

A lattice L is a discrete 'vector space' over $\mathbb{Z}$.

Lattices

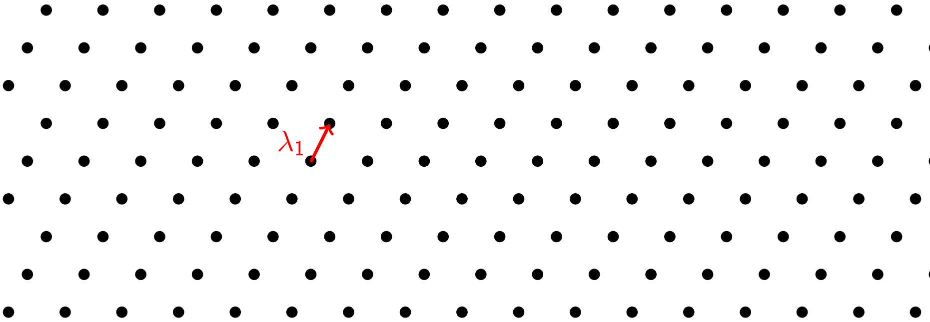


Lattice

A lattice L is a discrete 'vector space' over $\mathbb{Z}$.
A basis of L is an invertible matrix B such that $L = \{Bx \mid x \in \mathbb{Z}^n\}$.

$\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$ and $\begin{pmatrix} 17 & 11 \\ 4 & 2 \end{pmatrix}$ are two bases of the above lattice.

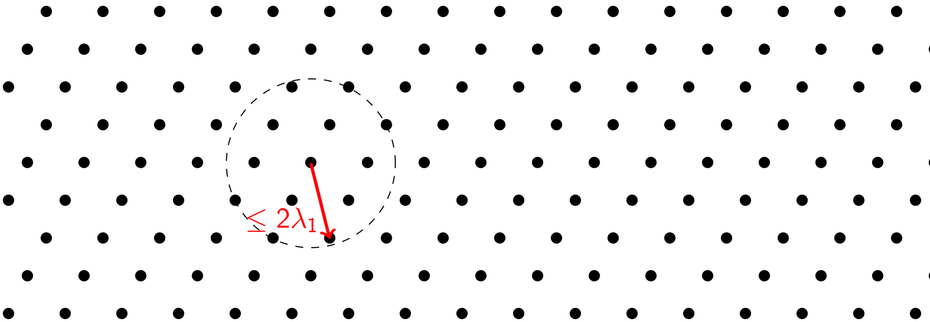
Lattices



Shortest Vector Problem (SVP)

Find a shortest (in Euclidean norm) non-zero vector.
Its Euclidean norm is denoted λ_1.

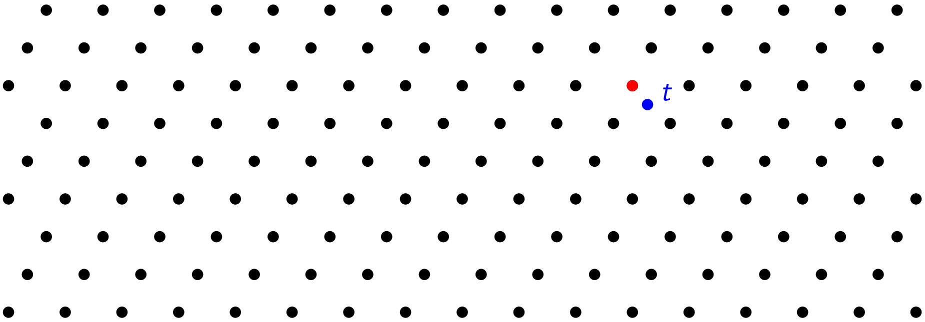
Lattices



Approximate Shortest Vector Problem (approx-SVP)

Find a short (in Euclidean norm) non-zero vector.
(e.g. of norm $\leq 2\lambda_1$).

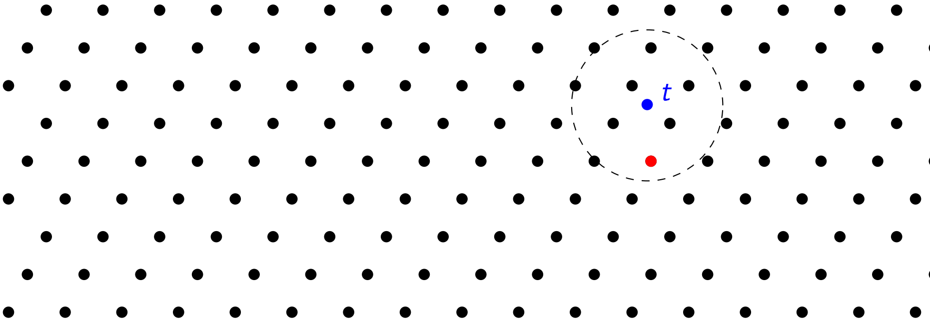
Lattices



Closest Vector Problem (CVP)

Given a target point t, find a point of the lattice closest to t.

Lattices



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Complexity of SVP/CVP

Applications

SVP and CVP in general lattices are conjectured to be hard to solve both quantumly and classically $\Rightarrow$ used in cryptography

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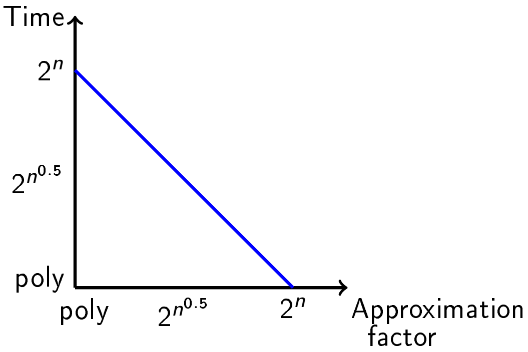
Best Time/Approximation trade-off for general lattices: BKZ algorithm



Structured lattices

Improve efficiency of lattice-based crypto using structured lattices, e.g.

$$M = \begin{pmatrix} a_1 & a_2 & \cdots & a_n \\ -a_n & a_1 & \cdots & a_{n-1} \\ \vdots & \ddots & \ddots & \vdots \\ -a_2 & -a_3 & \dots & a_1 \end{pmatrix}$$

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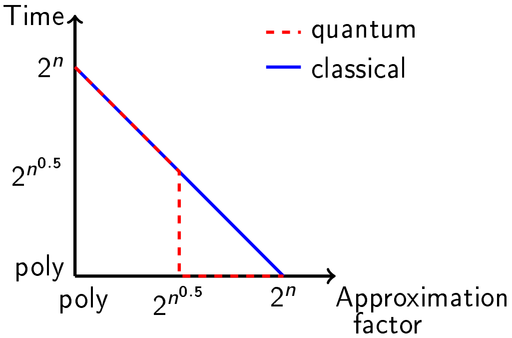
Is approx-SVP still hard when restricted to ideal lattices?

SVP in ideal lattices

[CDPR16,CDW17]: Better than BKZ in the quantum setting

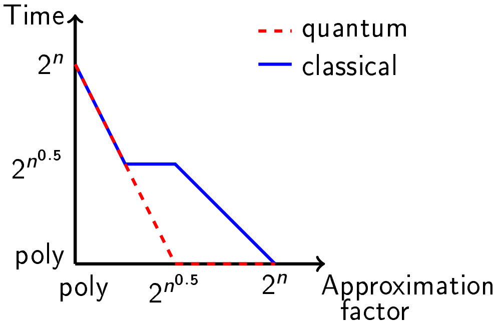


- Heuristic
- For prime power cyclotomic fields

[CDPR16] R. Cramer, L. Ducas, C. Peikert and O. Regev. Recovering Short Generators of Principal Ideals in Cyclotomic Rings, Eurocrypt.

[CDW17] R. Cramer, L. Ducas, B. Wesolowski. Short Stickelberger Class Relations and Application to Ideal-SVP, Eurocrypt.

This work



- Heuristic
- Pre-processing $2^{O(n)}$, independent of the choice of the ideal (non-uniform algorithm).

Outline of the talk

1. Definitions and objective
2. The CDPR algorithm
3. This work
4. Extensions and conclusion

First definitions

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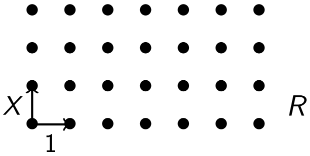
- Units: $R^\times = \{a \in R \mid \exists b \in R, ab = 1\}$
  - e.g. $\mathbb{Z}^\times = \{-1, 1\}$
- Principal ideals: $\langle g \rangle = \{gr \mid r \in R\}$ (i.e. all multiples of g)
  - e.g. $\langle 2 \rangle = \{\text{even numbers}\}$ in $\mathbb{Z}$
  - g is called a generator of $\langle g \rangle$
  - The generators of $\langle g \rangle$ are exactly the ug for $u \in R^\times$

Why is $\langle g \rangle$ a lattice?

$R \simeq \mathbb{Z}^n$

$$R = \mathbb{Z}[X]/(X^n + 1) \to \mathbb{Z}^n$$
$$r = r_0 + r_1 X + \cdots + r_{n-1} X^{n-1} \mapsto (r_0, r_1, \ldots, r_{n-1})$$



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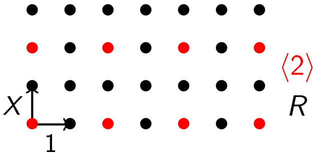
$$\begin{cases} \langle g \rangle \subseteq R \simeq \mathbb{Z}^n \\ \text{stable by `$+$' and `-'} \end{cases} \Rightarrow \text{ lattice}$$



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Find $r \in \langle g \rangle$ such that $\|r\| \le 2^{\widetilde{O}(n^\alpha)} \cdot \lambda_1$.

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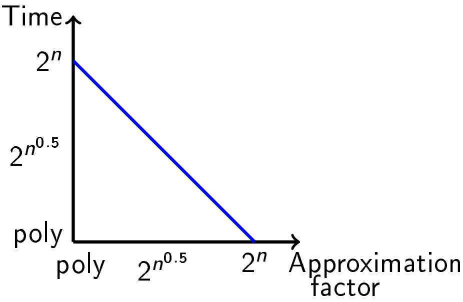
BKZ algorithm can do it in time $2^{O(n^{1-\alpha})}$, can we do better?



Outline of the talk

Main idea of the CDPR algorithm (on an idea of [CGS14])

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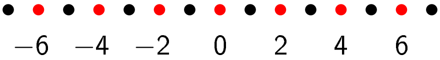

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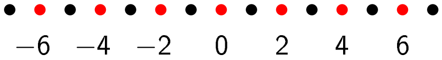

For larger n: one of the generators is somehow small

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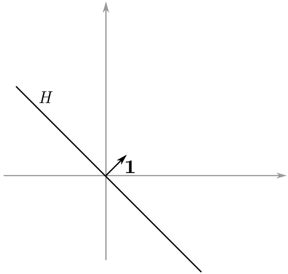
The Log space

Log : $R \to \mathbb{R}^n$ (somehow generalising log to R)

Let $\mathbf{1} = (1, \cdots, 1)$ and $H = \mathbf{1}^{\perp}$.



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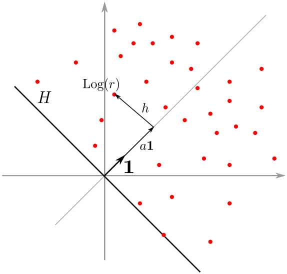

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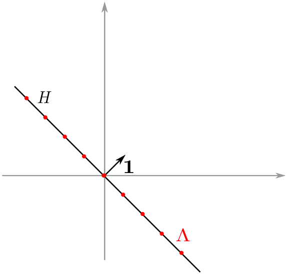

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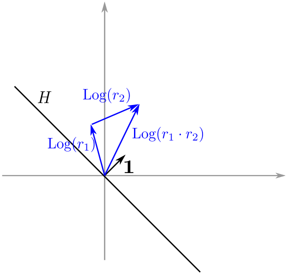

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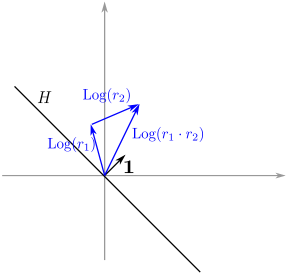
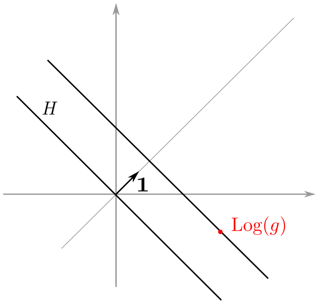
- $\|r\| \simeq 2^{\|\text{Log}\, r\|_\infty}$



The CDPR algorithm

What does $\mathrm{Log}\langle g\rangle$ look like?



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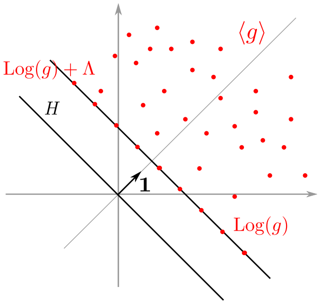

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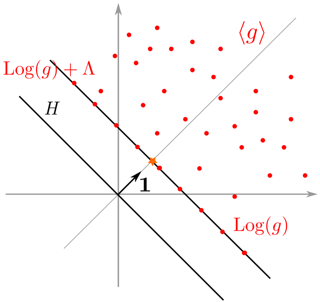

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The CDPR Algorithm:

- Find a generator g_1 of $\langle g \rangle$.
  - [BS16]: quantum time $\mathrm{poly}(n)$
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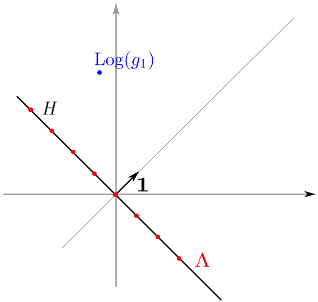

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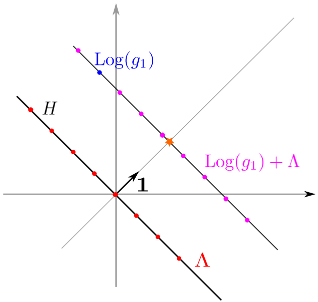

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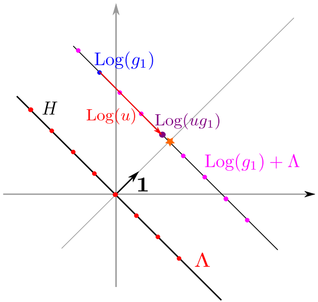

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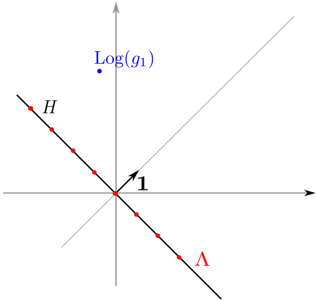

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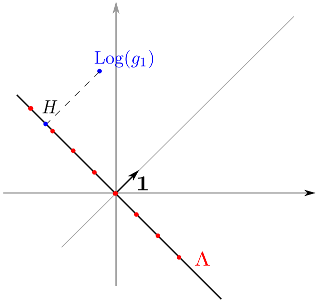

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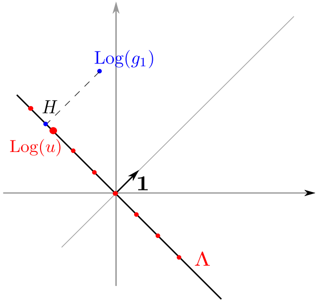

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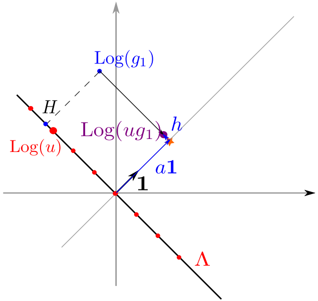

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- Find a generator g_1 of $\langle g \rangle$.
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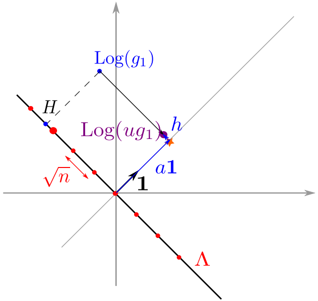

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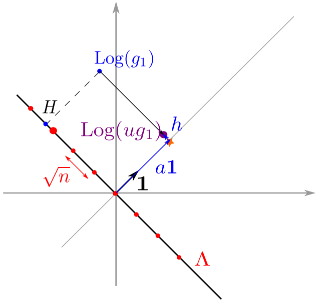

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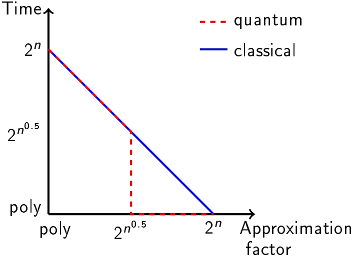

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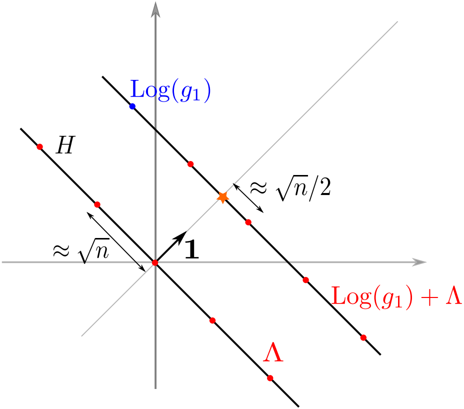
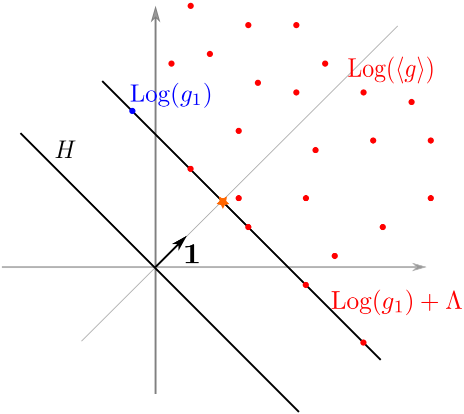
Outline of the talk

Idea



$\mathrm{Log}(g_1)$
H
$\approx \sqrt{n}/2$
$\approx \sqrt{n}$
$\mathbf{1}$
$\mathrm{Log}(g_1) + \Lambda$
Λ

Idea



$\mathrm{Log}(\langle g\rangle)$
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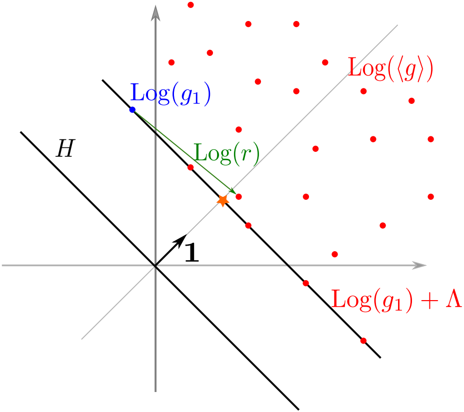
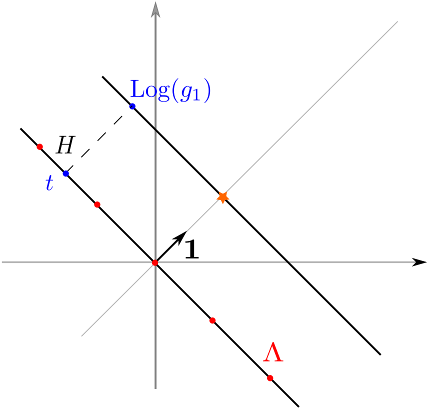


Important

$\mathrm{Log}\, r = h + a\mathbf{1}$ with a small (and $h \in H$).

Idea



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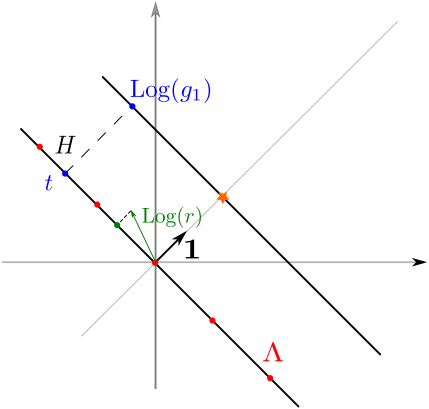

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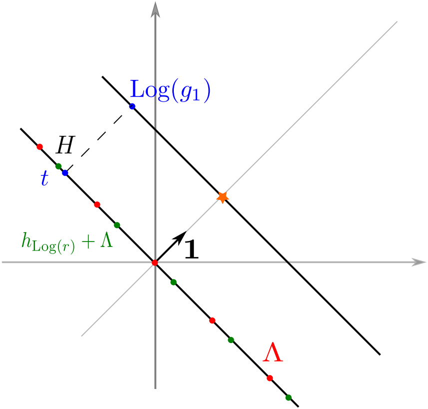

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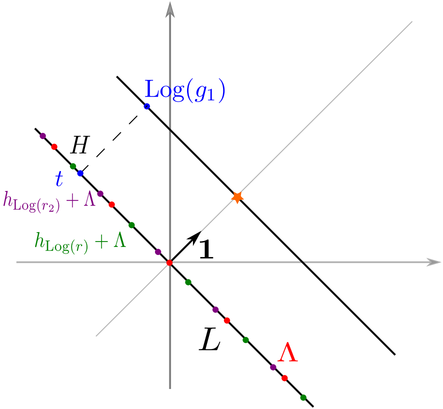

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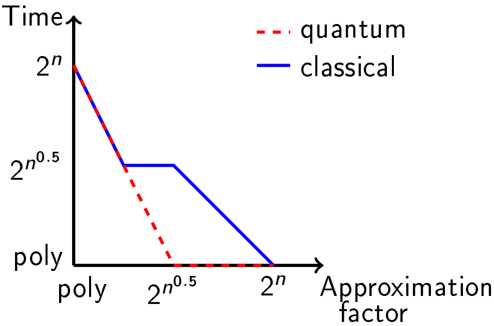
[Laa16]:
- Find $s \in L$ such that $\|s - t\| = \widetilde{O}(n^{\alpha})$
- Time:
  - $2^{\widetilde{O}(n^{1-2\alpha})}$ (query)
  - $+\ 2^{O(n)}$ (pre-processing)

[Laa16] T. Laarhoven. Finding closest lattice vectors using approximate Voronoi cells. SAC.

Conclusion

Approximation	Query time	Pre-processing
$2^{\widetilde{O}(n^{\alpha})}$	$2^{\widetilde{O}(n^{1-2\alpha})} + (\mathrm{poly}(n)$ or $2^{\widetilde{O}(\sqrt{n})})$	$2^{O(n)}$



$+2^{O(n)}$ Pre-processing / Non-uniform algorithm

Outline of the talk

Extensions

We can extend the algorithm to

- Non principal ideals

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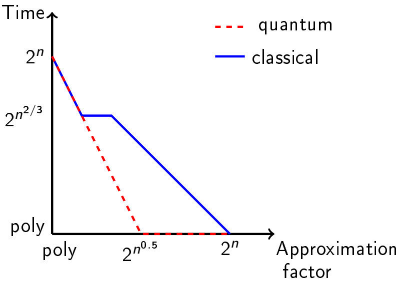
- Other number fields



Extension to any ideal

Ideal

An ideal is $I = \{ar_1 + br_2,\ r_1, r_2 \in R\}$ for some $a, b \in R$
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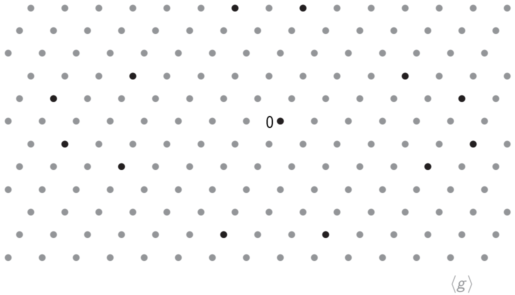

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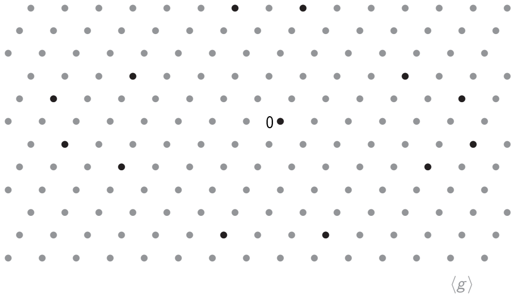

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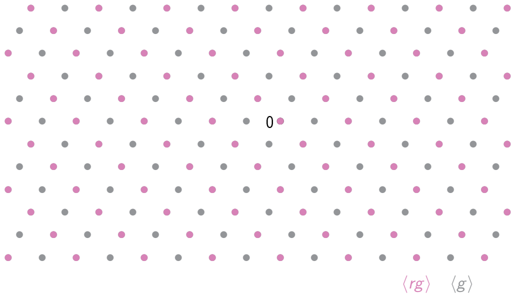

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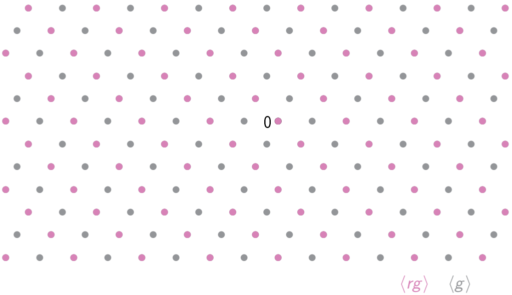

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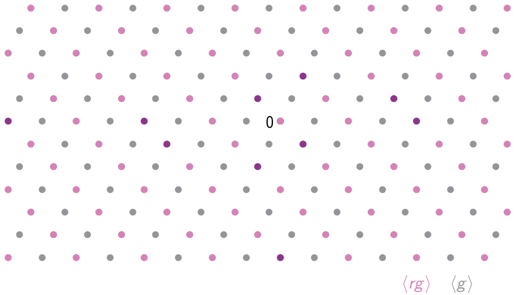

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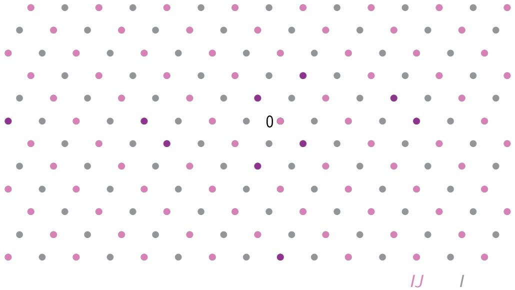

Extension to any ideal

- I has no generator (not principal)
- Multiply by some small ideal J
  - IJ sublattice of I
  - not much smaller
  - principal
  - with a small generator •

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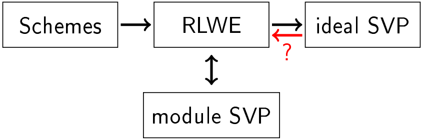

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