Overview of attacks on ideal lattices

Alice Pellet-Mary

CNRS and Université de Bordeaux

talk at ANSSI

Outline of the talk

1. First definitions

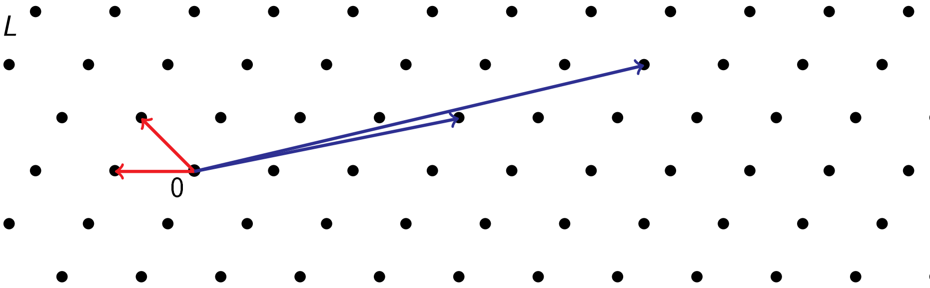
2. Context

3. State-of-the-art for ideal-SVP

4. Techniques

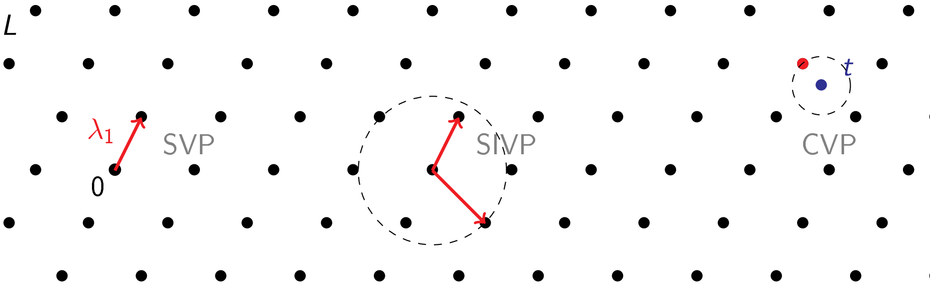
First definitions

Lattices



- $L = \{Bx \mid x \in \mathbb{Z}^n\}$ is a lattice
- $B \in \mathrm{GL}_n(\mathbb{R})$ is a basis
- n is the dimension of L

Algorithmic problems

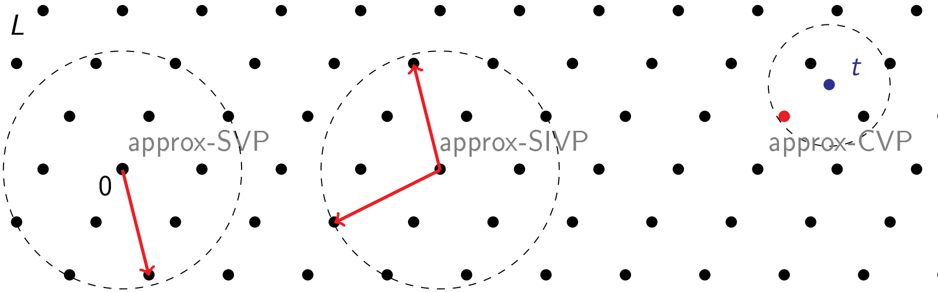


SVP
shortest vector problem

SIVP
shortest independent vector problem

CVP
closest vector problem

Algorithmic problems



approx-SVP
shortest vector problem

approx-SIVP
shortest independent vector problem

approx-CVP
closest vector problem

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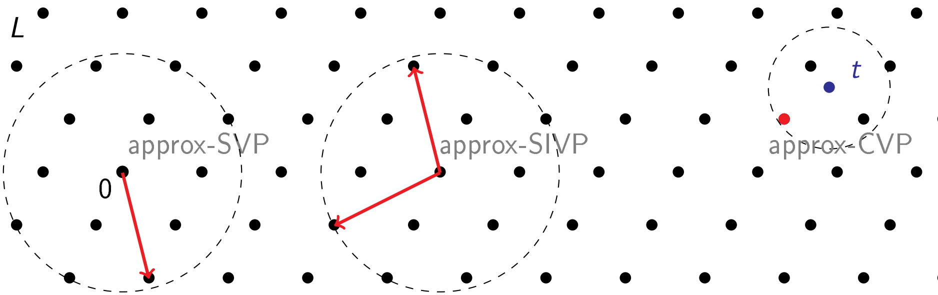


approx-SVP
shortest vector problem

approx-SIVP
shortest independent vector problem

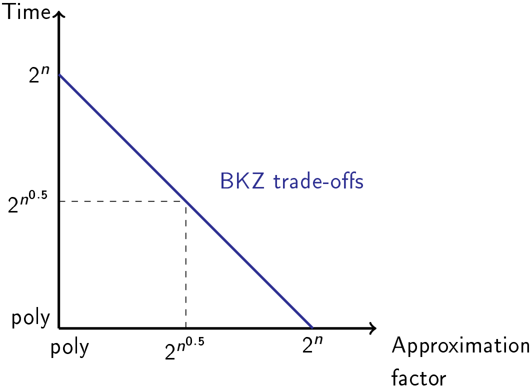
approx-CVP
closest vector problem

Supposedly hard to solve when n is large (input: a bad basis of L)

- even with a quantum computer
- even with a small approximation factor ($\mathrm{poly}(n)$)

Hardness of SVP and CVP

Best Time/Approximation trade-off for SVP, CVP (even quantumly):
BKZ algorithm [Sch87,SE94]



[Sch87] C.-P. Schnorr. A hierarchy of polynomial time lattice basis reduction algorithms. TCS.

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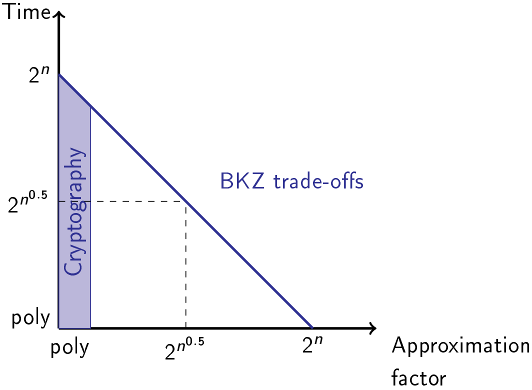

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Structured lattices

$$M_a = \begin{pmatrix} a_1 & -a_n & \cdots & -a_2 \\ a_2 & a_1 & \cdots & -a_3 \\ \vdots & \ddots & \ddots & \vdots \\ a_n & a_{n-1} & \dots & a_1 \end{pmatrix}$$

basis of a special case of
ideal lattice

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$$\begin{array}{ccc} n & \begin{pmatrix} M_{a_{11}} & M_{a_{12}} & \cdots & M_{a_{1m}} \\ M_{a_{21}} & M_{a_{22}} & \cdots & M_{a_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ M_{a_{m1}} & M_{a_{m2}} & \cdots & M_{a_{mm}} \end{pmatrix} & nm \end{array}$$

basis of a special case of
module lattice
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basis of a special case of
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ideal-SVP = SVP restricted to ideal lattices
module-SVP = SVP restricted to module lattices
$\Rightarrow$ hardness of these restricted problems much less understood than SVP

Context

Standard lattice-based problems

LWE

- post-quantum
- equivalent to worst-case SIVP in unstructured lattice
- not super efficient

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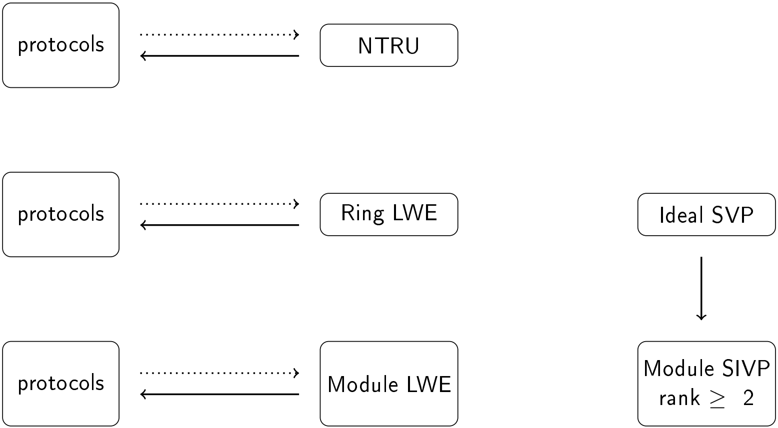
RLWE / Module-LWE / NTRU

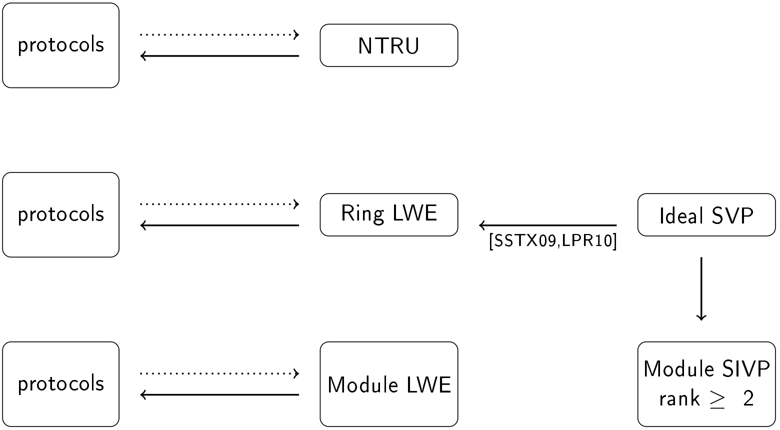
- post-quantum
- efficient
- how do they compare to structured lattice problems?

Reductions



⚠ Arrows may not all compose (different parameters) ⚠

Reductions



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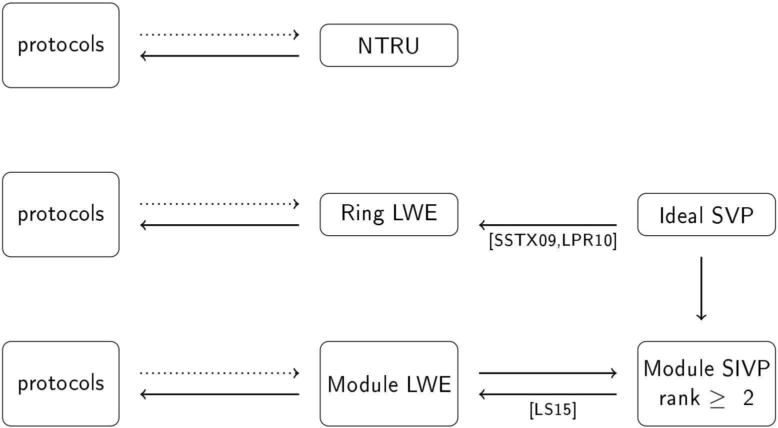
[SSTX09] Stehlé, Steinfeld, Tanaka, Xagawa. Efficient public key encryption based on ideal lattices. Asiacrypt.

[SSTX09] Lyubashevsky, Peikert, Regev. On ideal lattices and learning with errors over rings. Eurocrypt.

Reductions



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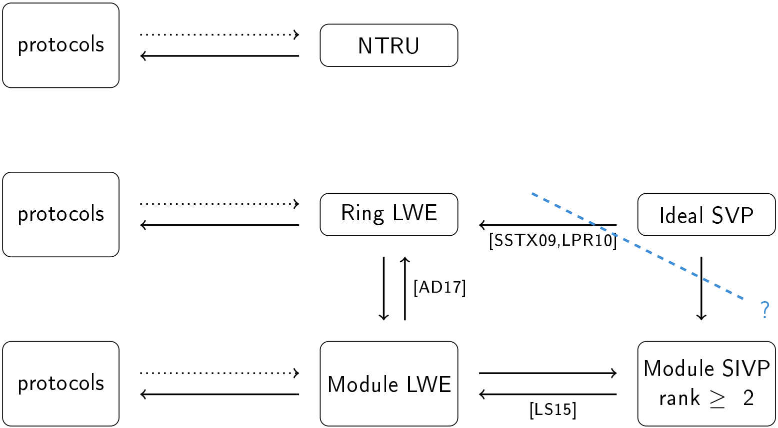
[LS15] Langlois, Stehlé. Worst-case to average-case reductions for module lattices. DCC.

Reductions



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[AD17] Albrecht, Deo. Large modulus ring-LWE ≥ module-LWE. Asiacrypt.

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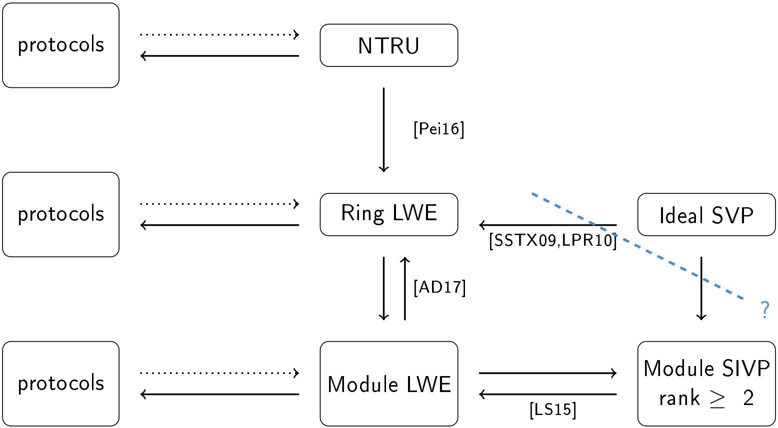

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[Pei16] Peikert. A decade of lattice cryptography. Foundations and Trends in TCS.

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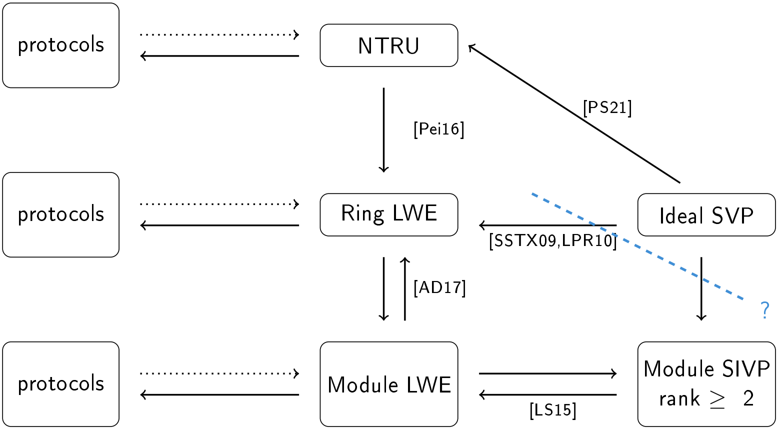

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[PS21] Pellet-Mary, Stehlé. On the hardness of the NTRU problem. Asiacrypt.

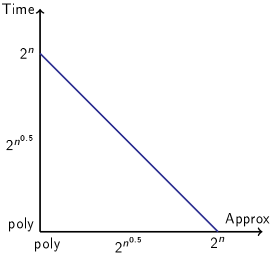
Summing up

breaking ideal-SVP $\not\Rightarrow$ breaking RLWE / module-LWE / NTRU

- as long as the attack does not generalize to rank ≥ 2, we are safe
- belief that there is a gap between rank 1 (ideals) and rank ≥ 2

State-of-the-art for ideal-SVP

How easy is ideal-SVP compared to SVP?



Unstructured lattices

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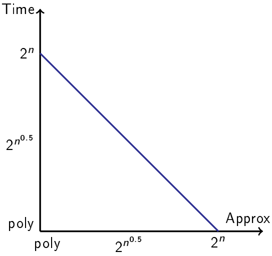

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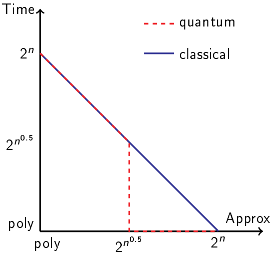
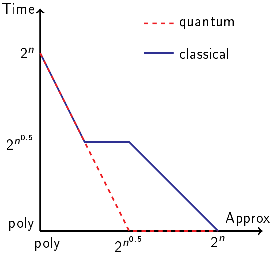


ideals lattices [CDW17]
(in cyclotomic fields)



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[PHS19] Pellet-Mary, Hanrot, Stehlé. Approx-SVP in ideal lattices with pre-processing. Eurocrypt.
[BR20] Bernard, Roux-Langlois. Twisted-PHS: using the product formula to solve approx-SVP in ideal lattices. AC.

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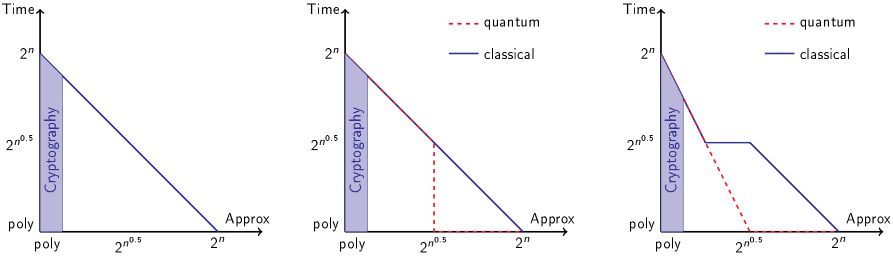

Unstructured lattices

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- almost no impact for crypto size params
- no reduction from RLWE to ideal-SVP

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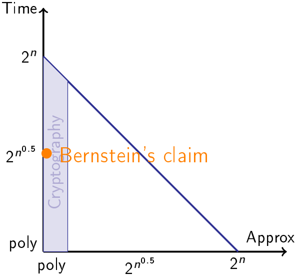
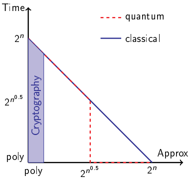
Bernstein's claim



Ideal lattices
(power-of-two cyclotomic)



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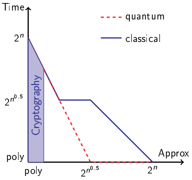

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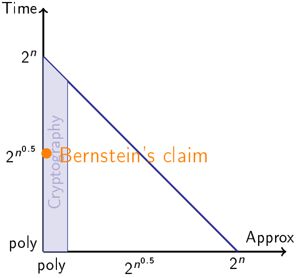

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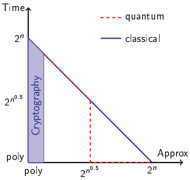

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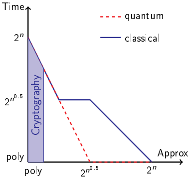

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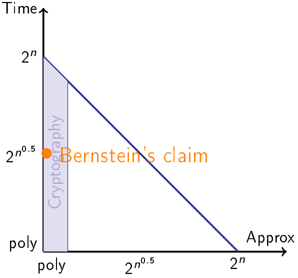

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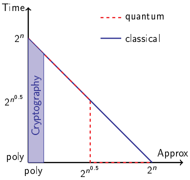
⚠ So far, no argument to support this claim



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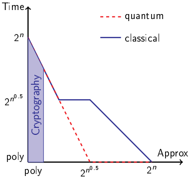

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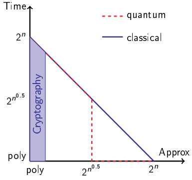
Concrete impact

When does [CDW17] starts out-performing BKZ?



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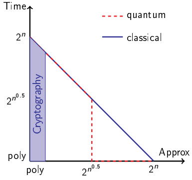
When does [CDW17] starts out-performing BKZ?

- For reasonable run-time (a few core-days):
  - dim $\gtrsim 5\,000$
- For NIST's weakest security requirement:
  - dim $\gtrsim 17\,000$



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- For reasonable run-time (a few core-days):
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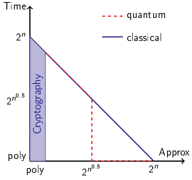
- For NIST's weakest security requirement:
  - dim $\gtrsim 17\,000$
- Dimension of NIST candidates:
  - dim ≈ 500 or $1\,000$



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Techniques

Math background

Notation

$K = \mathbb{Q}[X]/(X^n + 1)$, with $n = 2^k$ (or any cyclotomic field)

$O_K = \mathbb{Z}[X]/(X^n + 1)$

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- Units: $O_K^\times = \{a \in O_K \mid \exists b \in O_K, ab = 1\}$

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$O_K = \mathbb{Z}[X]/(X^n + 1)$

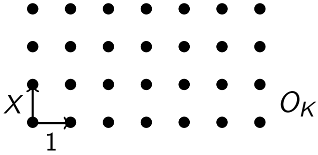
- Units: $O_K^\times = \{a \in O_K \mid \exists b \in O_K, ab = 1\}$
- Principal ideals: $\langle g \rangle = \{gr \mid r \in O_K\}$
  - g is a generator of $\langle g \rangle$
  - $\{$ generators of $\langle g \rangle$ $\} = \{gu \mid u \in O_K^\times\}$

Why is $\langle g \rangle$ a lattice?

O_K is a lattice

$$O_K = \mathbb{Z}[X]/(X^n+1) \rightarrow \mathbb{R}^n$$

$$r(X) = \sum_{i=0}^{n-1} r_i X^i \mapsto (r_0, r_1, \ldots, r_{n-1})$$



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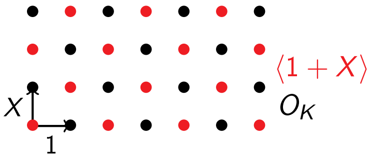
$$\begin{cases} \langle g \rangle \subseteq O_K \simeq \mathbb{Z}^n \\ \text{stable by '+' and '−'} \end{cases} \Rightarrow \text{ideal lattice}$$



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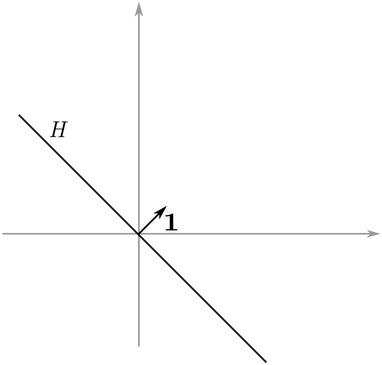
The Log space

Log : $O_K \to \mathbb{R}^n$ (take the log of every coordinate)

Let $1 = (1, \cdots, 1)$ and $H = 1^{\perp}$.



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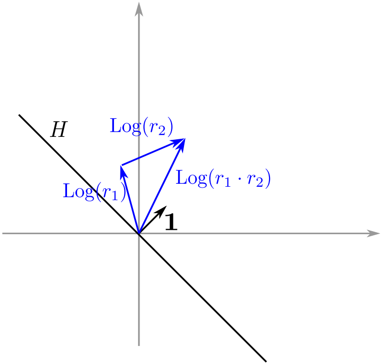
Properties ($r \in O_K$)

$\operatorname{Log} r = h + a \cdot \mathbf{1}$, with $h \in H$

- $\operatorname{Log}(r_1 \cdot r_2) = \operatorname{Log}(r_1) + \operatorname{Log}(r_2)$



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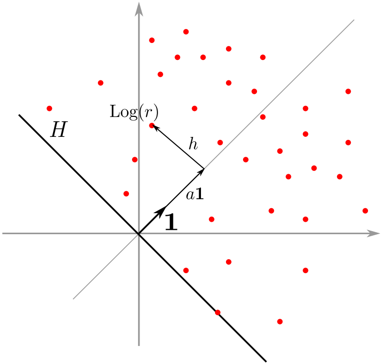

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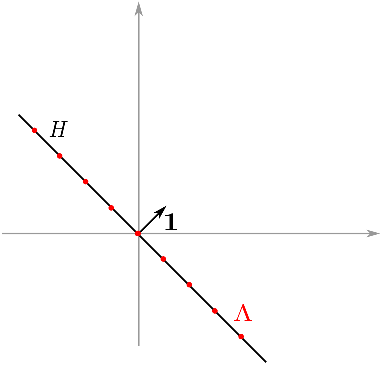

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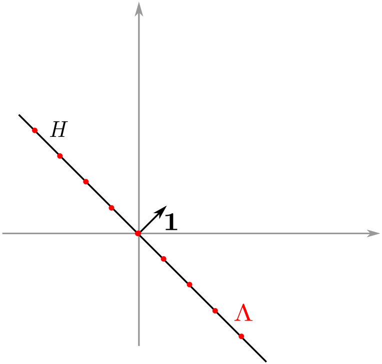
- $a \geq 0$
- $a = 0$ iff r is a unit

The Log space

$\mathrm{Log} : O_K \to \mathbb{R}^n$ (take the log of every coordinate)

Let $1 = (1, \cdots, 1)$ and $H = 1^{\perp}$.



Properties ($r \in O_K$)

$\mathrm{Log}\, r = h + a \cdot 1$, with $h \in H$

- $\mathrm{Log}(r_1 \cdot r_2) = \mathrm{Log}(r_1) + \mathrm{Log}(r_2)$
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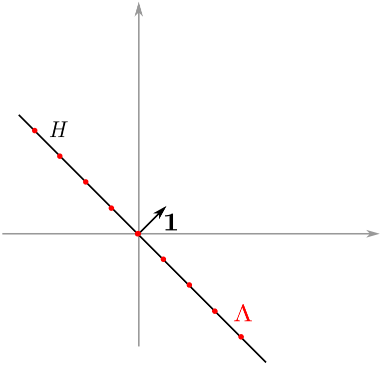

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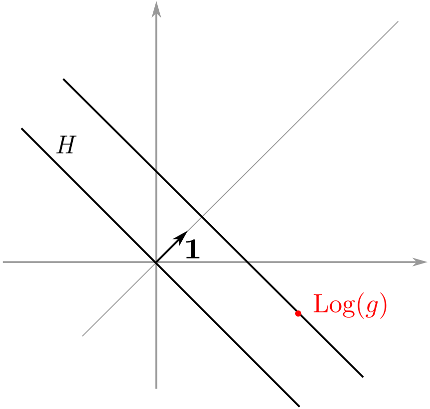
The basic algorithm [CGS14,CDPR16]

What does $\mathrm{Log}\langle g\rangle$ look like?



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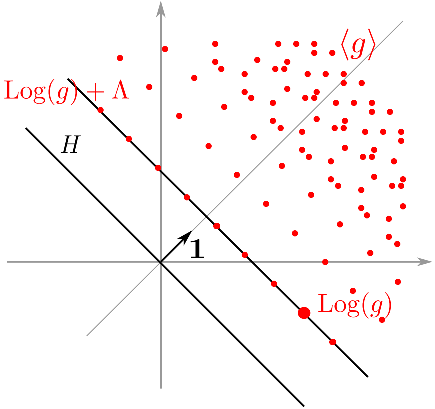

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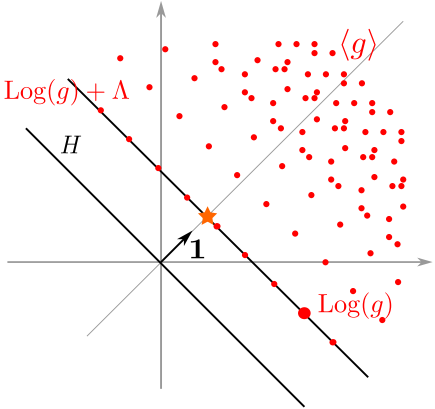

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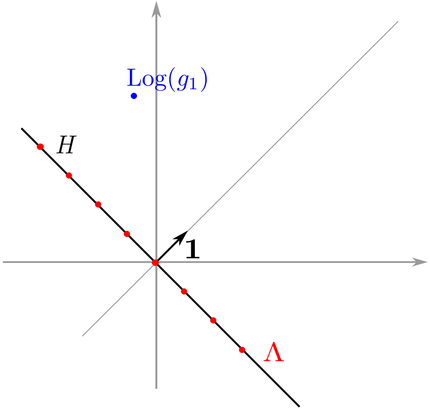

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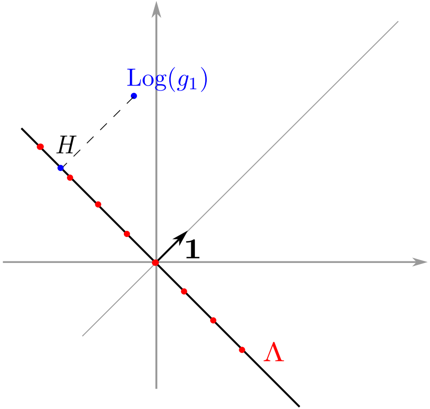

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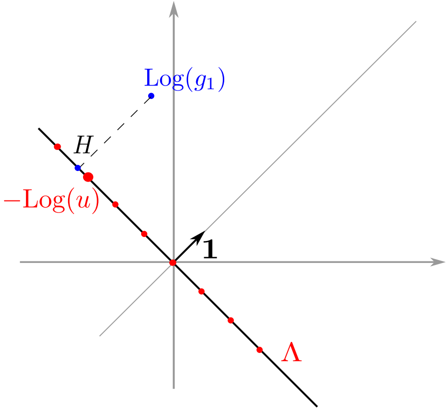

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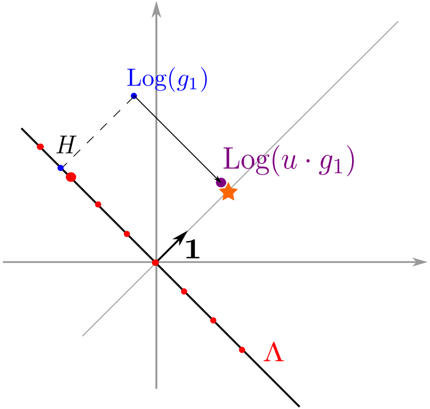

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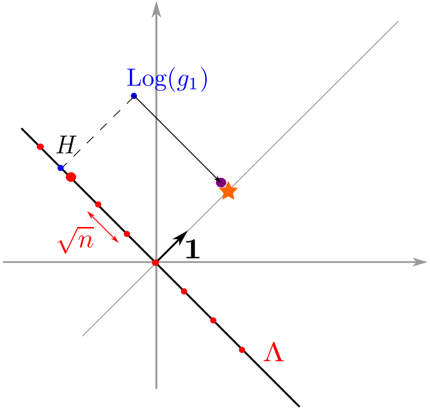

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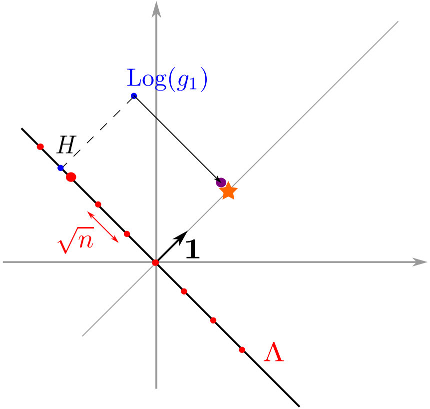

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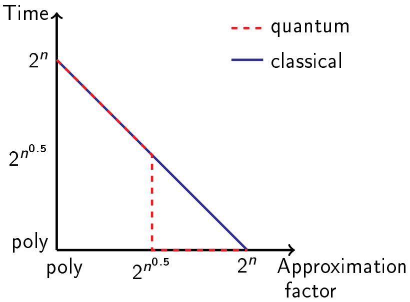

- Heuristic
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− we don't know a good basis of the Log-S-unit lattice
  - need to pre-compute it (time $2^{O(n)}$)
  - even with the best basis possible,
    we can only solve CVP with approx $O(\sqrt{n})$ in poly time
    $\Rightarrow$ still $2^{O(\sqrt{n})}$ approx-SVP in poly time

Conclusion

Attacks on module lattices

Not much:

- BKZ algorithm for modules [MS20]
  - does not outperform BKZ, but the algo uses only modules (no unstructured lattices)

[MS20] Mukherjee and Stephens-Davidowitz. Lattice reduction for modules, or how to reduce module-SVP to module-SVP. Crypto.

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- Algorithm for SVP in rank-2 modules [LPSW19]
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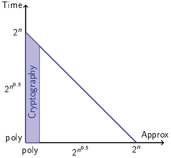
[LPSW19] Lee, Pellet-Mary, Stehlé, Wallet. An LLL algorithm for module lattices. Asiacrypt.

Conclusion



Unstructured lattices
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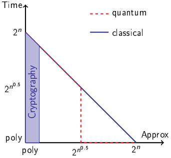

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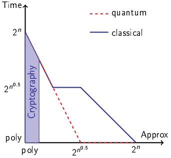

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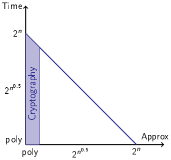

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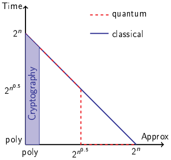

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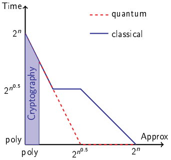

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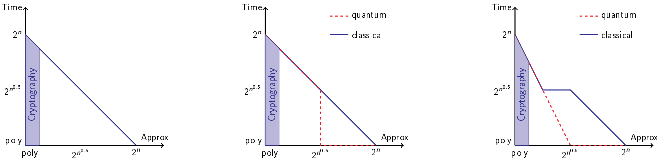

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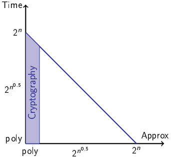

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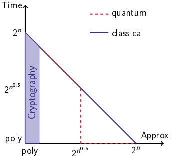

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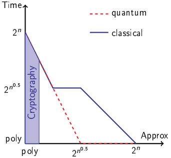

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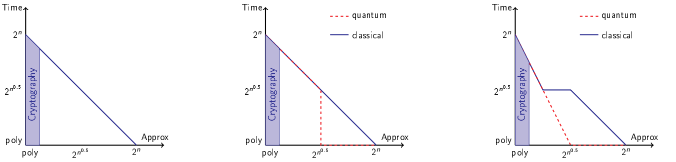

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